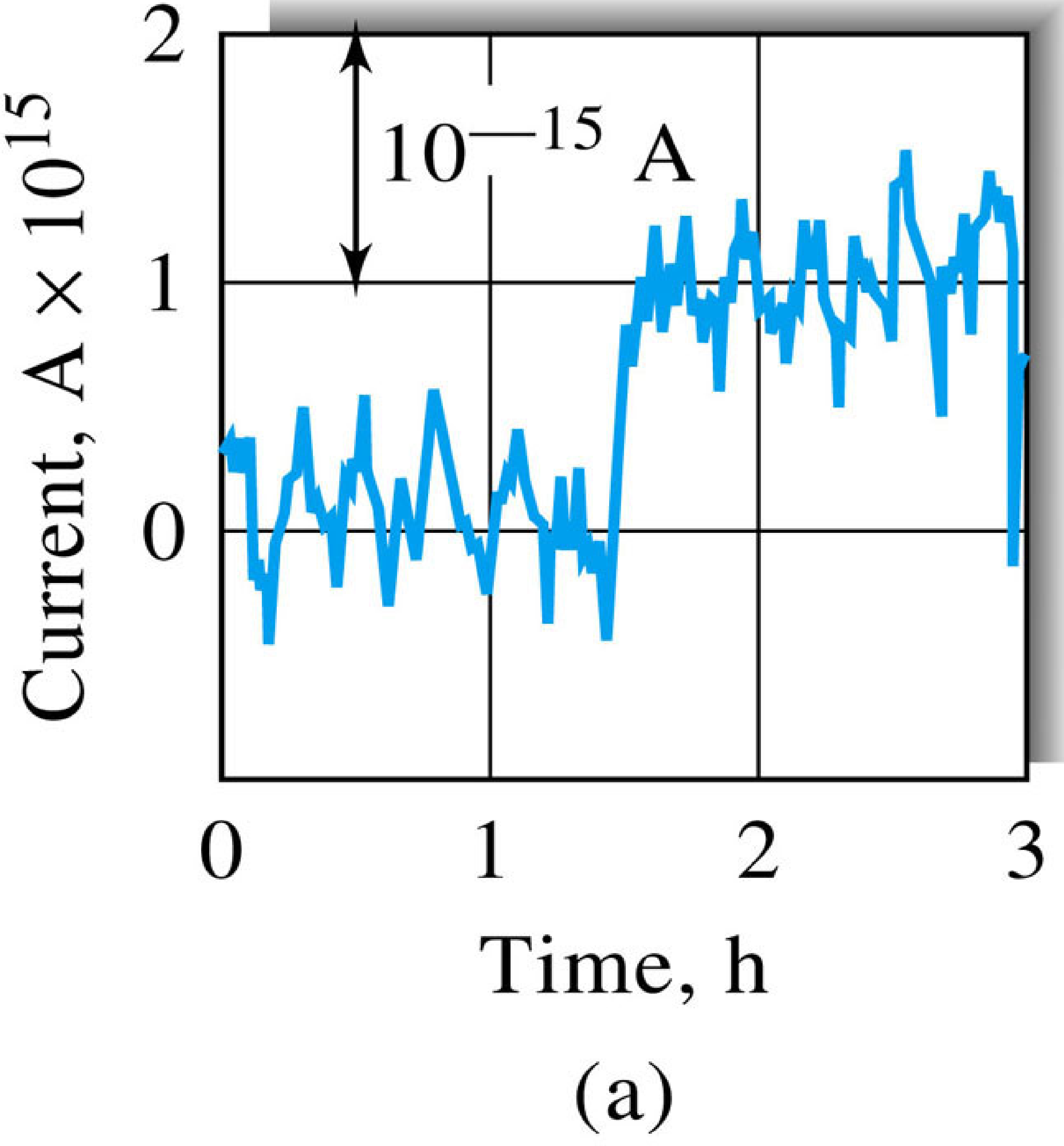
The Signal and the Noise

- Signal to Noise Ratio
- Types of Noise
- Signal to Noise Ratio Enhancement
  - Signal Averaging
  - Lock-in

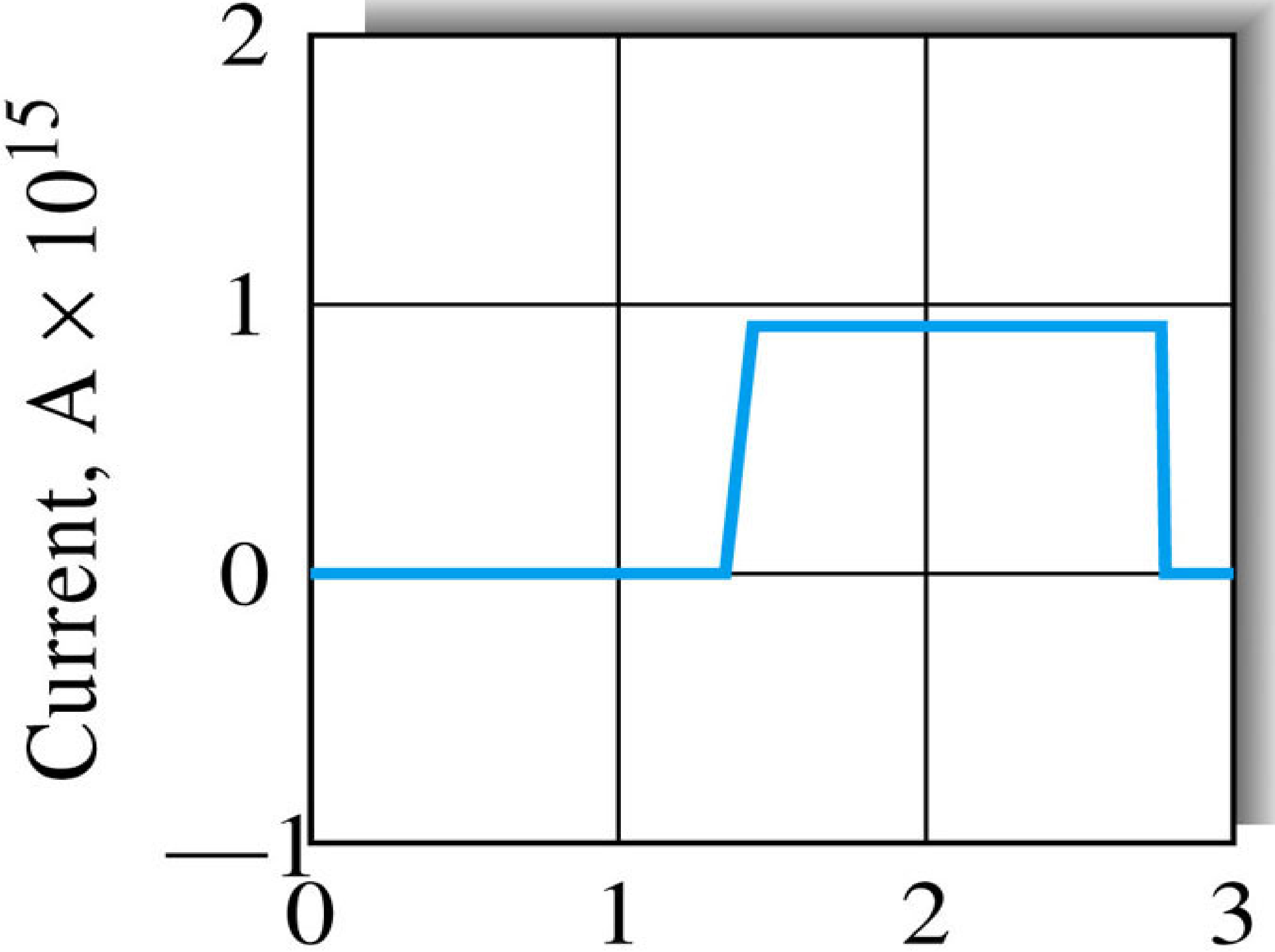
Signal-Noise Ratio

- The signal is what you are measuring that is the result of the presence of your analyte
- Noise is extraneous information that can interfere with or alter the signal.
- It can not be completely eliminated, but hopefully reduced!
  - Noise is considered random!
    - indeterminate

SIGNAL VS. NOISE



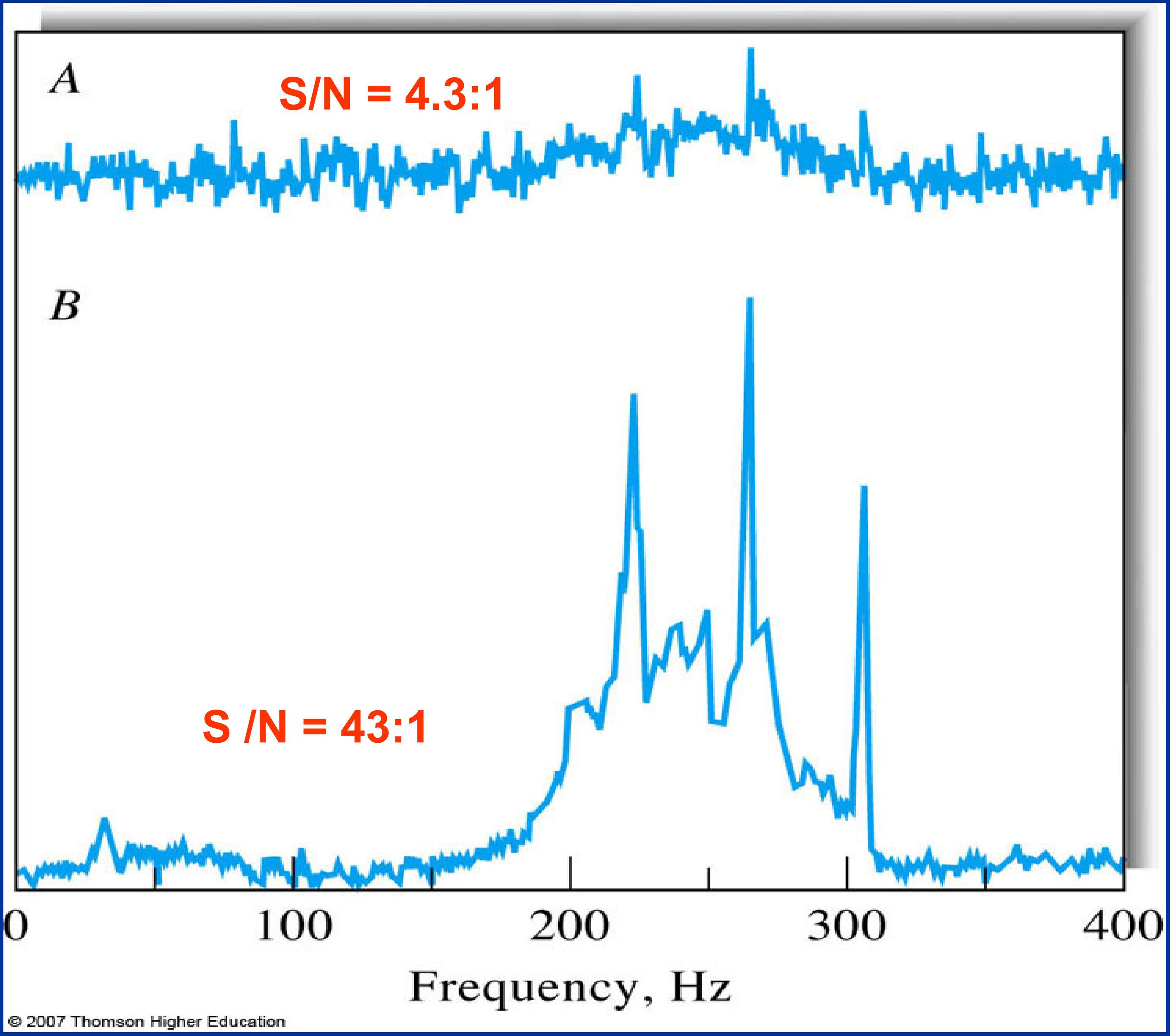
SIGNAL VS. NOISE



(b)

- **Since noise can not be eliminated (it is random), we are more interested in the S/N ratio than the intensity of the noise**

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\text{mean}}{\text{standard deviation}} = \frac{\overline{X}}{s}$$


A
S/N = 4.3:1
B
S /N = 43:1
0
100
200
300
400
Frequency, Hz
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S/N Objective?

- Reduce as much of the noise as possible by carefully controlling conditions
  - Temperature, power supply variations, etc. etc. etc.
- Increase the signal to noise ratio!
  - More signal vs. noise means a lower STDEV!
    - More precise measurement
  - Lower STDEV means a better LOD
    - Lower limits of detection
- A S/N ratio of 3 is usually the minimum that is acceptable.

Types of noise

Signal processing noise can be classified by its statistical properties (sometimes called the "color" of the noise) and by how it modifies the intended signal:

- **Additive** noise, gets added to the unintended signal
  - **Power-law noise**
  - White noise
    - Additive white Gaussian noise
  - Pink noise
    - Flicker noise, with 1/f power spectrum
    - Brown noise or Brownian noise, with $1/f^2$ power spectrum
  - Black noise
  - Gaussian noise
  - Contaminated Gaussian noise, whose PDF is a linear mixture of Gaussian PDFs
  - Cauchy noise
- **Multiplicative** noise, multiplies or modulates the intended signal
- **Quantization** error, due to conversion from continuous to discrete values
- **Poisson noise**, typical of signals that are rates of discrete events
  - Shot noise, e.g. caused by static electricity discharge
- Transient noise, a short pulse followed by decaying oscillations
- Burst noise, powerful but only during short intervals
- Phase noise, random time shifts in a signal

Stationary processes

- In a generic random process, all statistical quantities are a function of *t*.
- If they are instead time-invariant, then the process is called strict sense stationary.
- Stationarity means that it is impossible to discriminate between a process and its translated version in time only by mean of statistical measures.
- If a process is stationary, then its PDF must satisfy *f(x,t)=f(x,t+k)*, that is the PDF does not change in time.
- As a consequence, its mean must be constant:

$$m_x(t)=\int_{-\infty}^{\infty} x f_X(x,t)\,dx=\int_{-\infty}^{\infty} x f_X(x)\,dx=m_x$$

- And similarly the power and the variance:
  - $P_X(t)=P_X$
  - $\sigma^2_X(t)=\sigma^2_X$

Ergodic processes

- To characterize a process, even if stationary, we should consider all the instances and their statistical features
- If its possible to use only one instance to fully charcaterize the process then the process is called "ergodic".
- An ergodic process satisfies:
-
-

$$m_x(t) = m_x = X_m = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x(n)$$

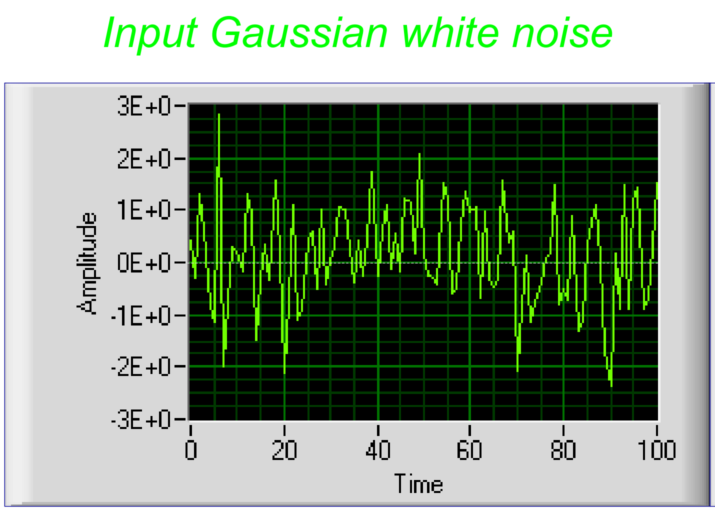
-
- That is the random variable X_m, obtained by averaging at single time all the possible instances, has a mean equal to m_x and null variance.
- In practice, the mean can be computed by using a "moving window"

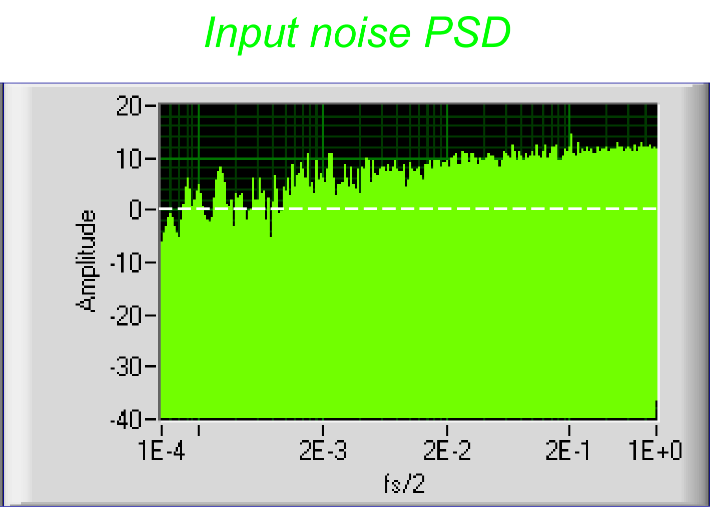
$$X_T = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \qquad\qquad X_m = \lim_{N \to \infty} X_T$$

ADDITIVE WHITE GAUSSIAN NOISE

- Noise in signal processing can mostly be considered a stationary random process.
  - Mean, variance and power are constant
  - Autocorrelation and autocovariance depend only on the difference between two time instants
- At a fixed *t, Noise(t)* is a random Gaussian variable
- If we assume that its autocorrelation is a Dirac delta in the origin, then its power spectral density will be constant
-
- Last, most noise add to the signals: *obs(t) = signal(t) + Noise(t)*
- AWGN = ADDITIVE WHITE GAUSSIAN NOISE

Example: A simulation of white noise





Instrumental Noise

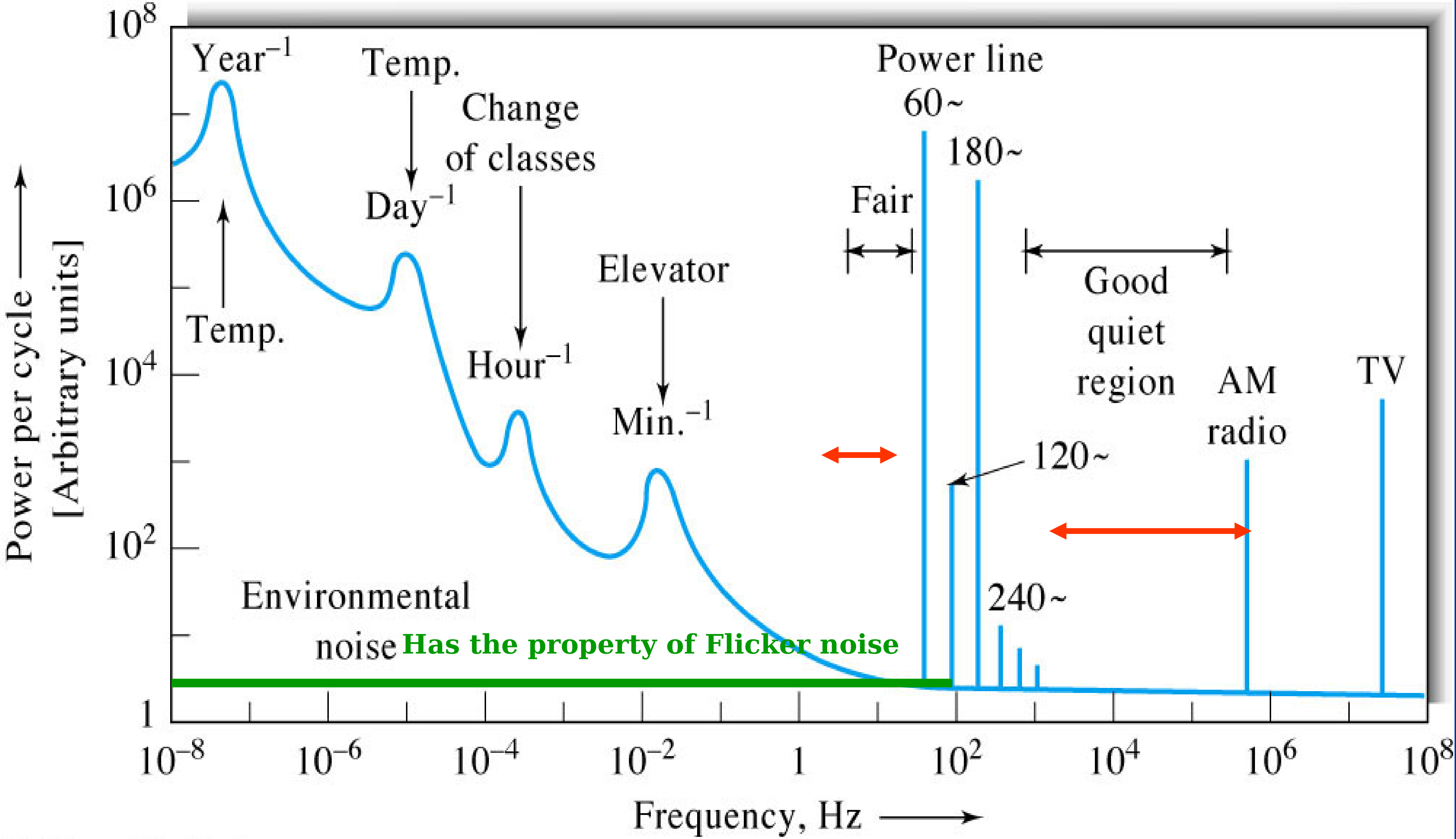
- **Thermal (Johnson) Noise:**
  - Thermal agitation of electrons affects their "smooth" flow.
  - Due to different velocities and movement of electrons in electrical components.
  - Dependent upon both temperature and the range of frequencies (frequency bandwidths) being utilized.
  - Can be reduced by reducing temperature of electrical components.
    - Eliminated at absolute zero.
  - Considered "white noise" occurs at all frequencies.
  - $\nu_{rms} = (4kTR\Delta f)^{1/2}$,

- **Shot Noise:**
  - **Occurs when electrons or charged particles cross junctions (different materials, vacuums, etc.)**
  - **Considered "white noise" occurs at all frequencies.**
  - **Due to the statistical variation of the flow of electrons (current) across some junction**
    - **Some of the electrons jump across the junction right away**
    - **Some of the electrons take their time jumping across the junction**
  - $i_{rms} = (2Ie\Delta f)^{1/2}$

- **Flicker Noise**
  - **Frequency dependent**
  - **Magnitude is inversely proportional to frequency**
  - **Significant at frequencies less than 100 Hz**
  - **Results in long-term drift in electronic components**
  - **Can be reduced by using resisters that are metallic, wire wound**

- **Environmental Noise: Room should be cold??**
  - **Unlimited possible sources**
  - **Can often be eliminated by eliminating the source**
    - **Other noise sources can not be eliminated!!!!!!**
  - **Methods of eliminating it…**
    - **Moving the instrument somewhere else**
    - **Isolating /conditioning the instruments power source**
    - **Controlling temperature in the room**
      - **Control expansion/contraction of components in instrument**
    - **Eliminating interferences**
      - **Stray light from open windows, panels on instrument**
      - **Turning off radios, TV's, other instruments**

NOISE SPECRUM



Pink noise

- Pink noise or 1⁄f noise is a signal or process with a frequency spectrum such that the power spectral density (energy or power per frequency interval) is inversely proportional to the frequency of the signal. In pink noise, each octave (halving/doubling in frequency) carries an equal amount of noise energy. The name arises from the pink appearance of visible light with this power spectrum.[1] This is in contrast with white noise which has equal intensity per frequency interval.

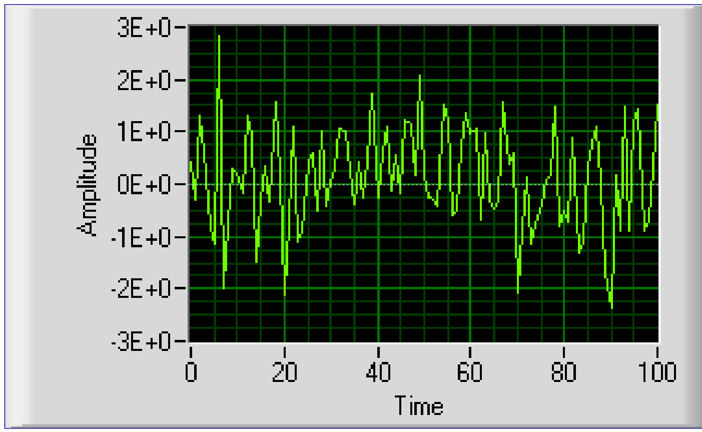
- Within the scientific literature the term pink noise is sometimes used a little more loosely to refer to any noise with a power spectral density of the form

- $S(f) \propto 1/f^{\alpha}$

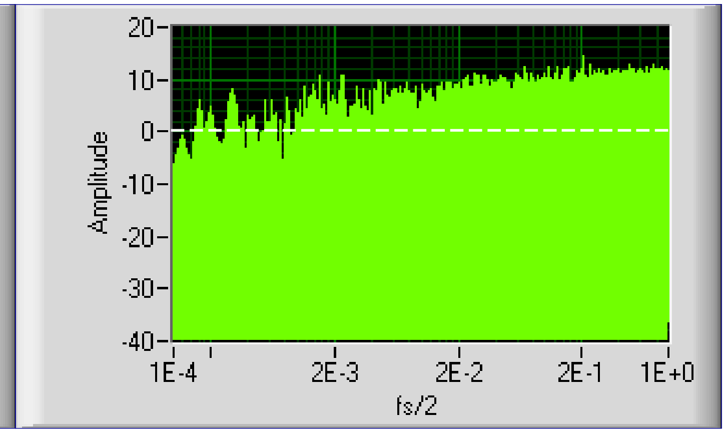
- where f is frequency, and $0 < \alpha < 2$, with exponent α usually close to 1.
- These pink-like noises occur widely in nature and are a source of considerable interest in many fields. The distinction between the noises with α near 1 and those with a broad range of α approximately corresponds to a much more basic distinction.

Example: A simulation of $1/f$ noise

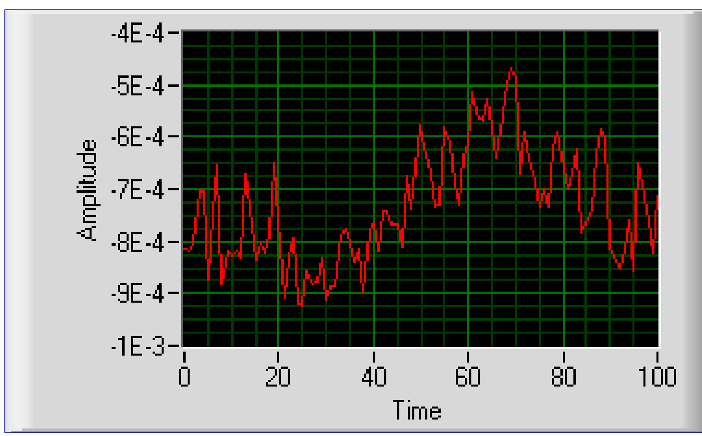
Input Gaussian white noise



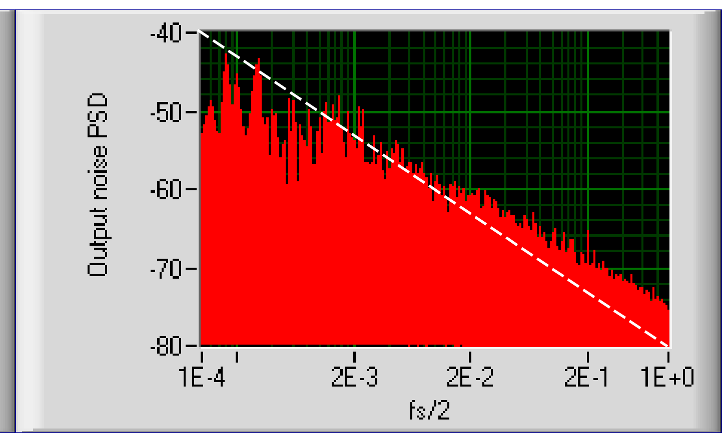
Input noise PSD



Output $1/f$ noise



Output noise PSD



Brownian noise

- Brownian noise, also known as Brown noise or red noise, is the kind of signal noise produced by Brownian motion, hence its alternative name of random walk noise.
- The term "Brown noise" comes not from the color, but after Robert Brown, the discoverer of Brownian motion.
- The term "red noise" comes from the "white noise"/"white light" analogy; red noise is strong in longer wavelengths, similar to the red end of the visible spectrum.
- Its spectral density is inversely proportional to f^2, meaning it has more energy at lower frequencies, even more so than pink noise. It decreases in power by 6 dB per octave (20 dB per decade)
- Strictly, Brownian motion has a Gaussian probability distribution, but "red noise" could apply to any signal with the $1/f^2$ frequency spectrum.

Brownian noise

- Brown noise can be produced by integrating white noise.
- That is, whereas (digital) white noise can be produced by randomly choosing each sample independently, Brown noise can be produced by adding a random offset to each sample to obtain the next one.
- A Brownian motion, also called a Wiener process, is obtained as the integral of a white noise signal:

$$W(t) = \int_0^t \frac{dW(\tau)}{d\tau} d\tau$$

- meaning that Brownian motion is the integral of the white noise dW(t), whose power spectral density is flat:

$$S_0 = \left| \mathcal{F}\left[\frac{dW(t)}{dt}\right](\omega) \right|^2 = \text{const.}$$

- where $\mathcal{F}$ denotes the Fourier transform, and S_0 is a constant. An important property of Fourier transform is that the derivative of any distribution transforms as

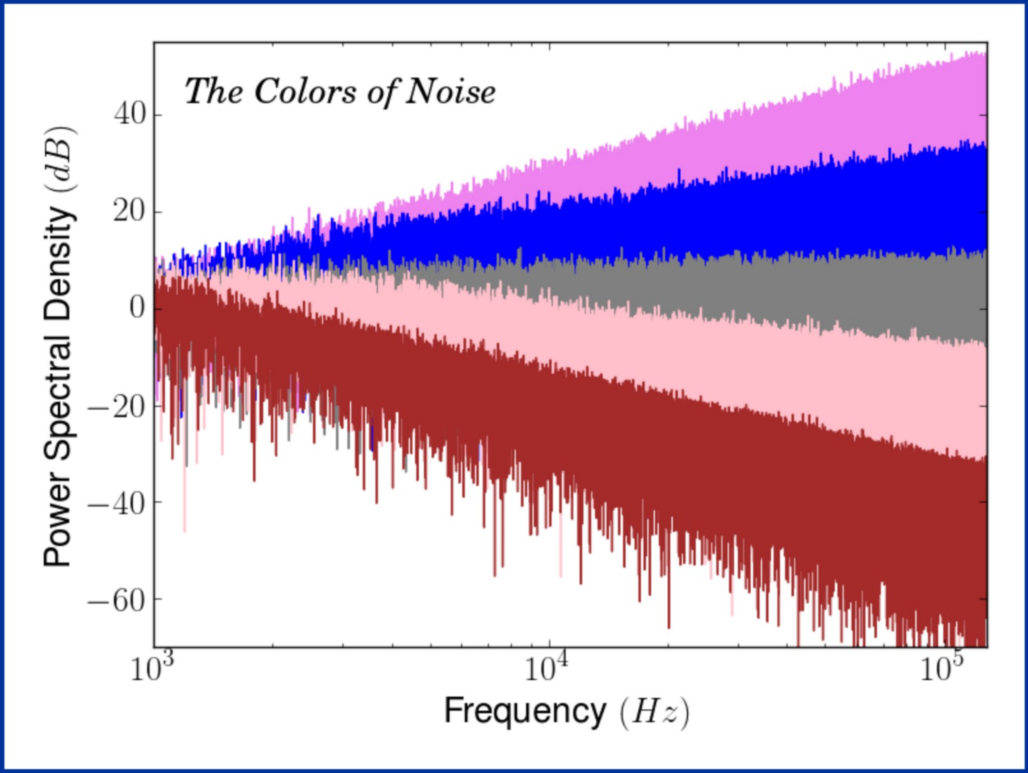
$$\mathcal{F}\left[\frac{dW(t)}{dt}\right](\omega) = i\omega\mathcal{F}[W(t)](\omega),$$

- from which we can conclude that the power spectrum of Brownian noise is

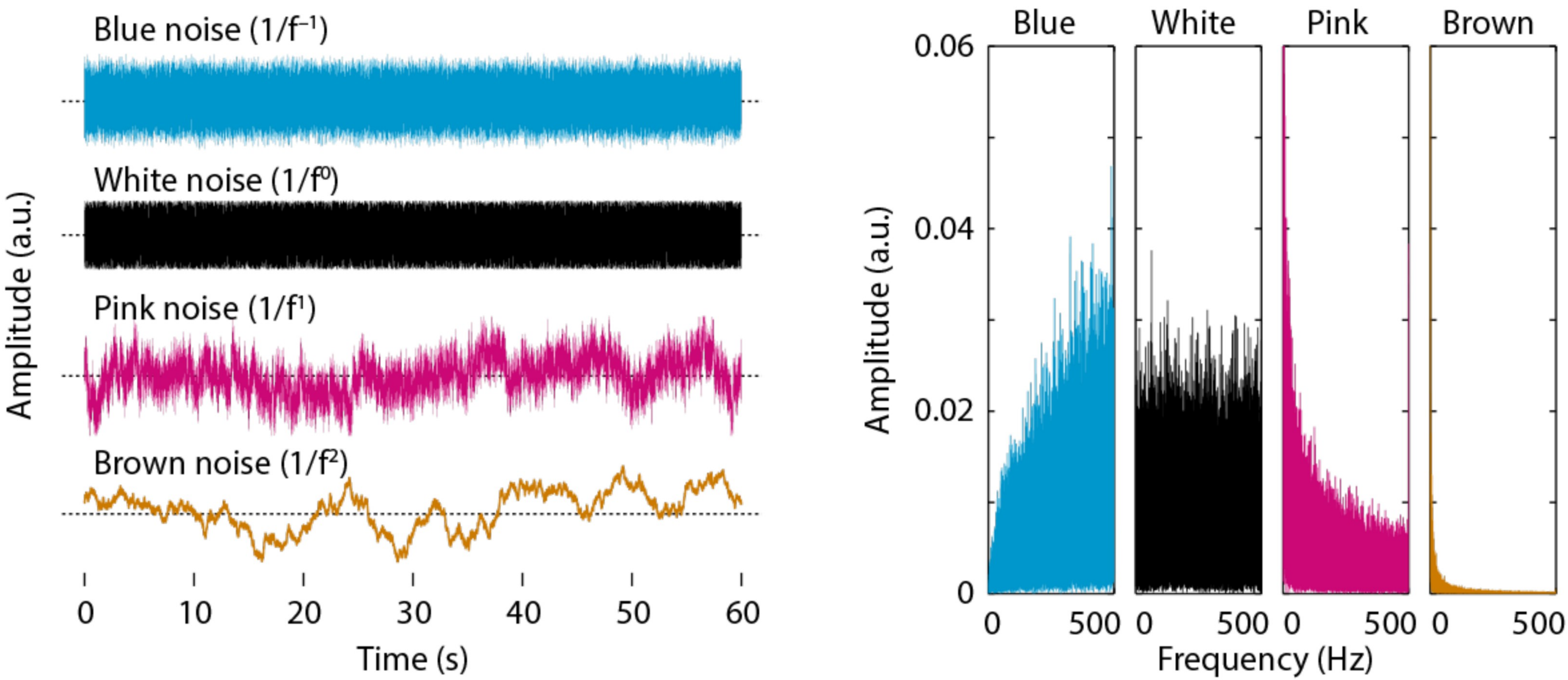
$$S(\omega) = \left|\mathcal{F}[W(t)](\omega)\right|^2 = \frac{S_0}{\omega^2}.$$

Power Law noise

- The color of noise refers to the power spectrum of a noise signal.
- Different colors of noise have significantly different properties.
- The practice of naming kinds of noise after colors started with white noise, a signal whose spectrum has equal power within any equal interval of frequencies.
- Many of these definitions assume a signal with components at all frequencies, with a power spectral density per unit of bandwidth proportional to $1/f^{\beta}$ and hence they are examples of power-law noise.
- For instance, the spectral density of white noise is flat ($\beta = 0$), while flicker or pink noise has $\beta = 1$, and Brownian noise has $\beta = 2$.



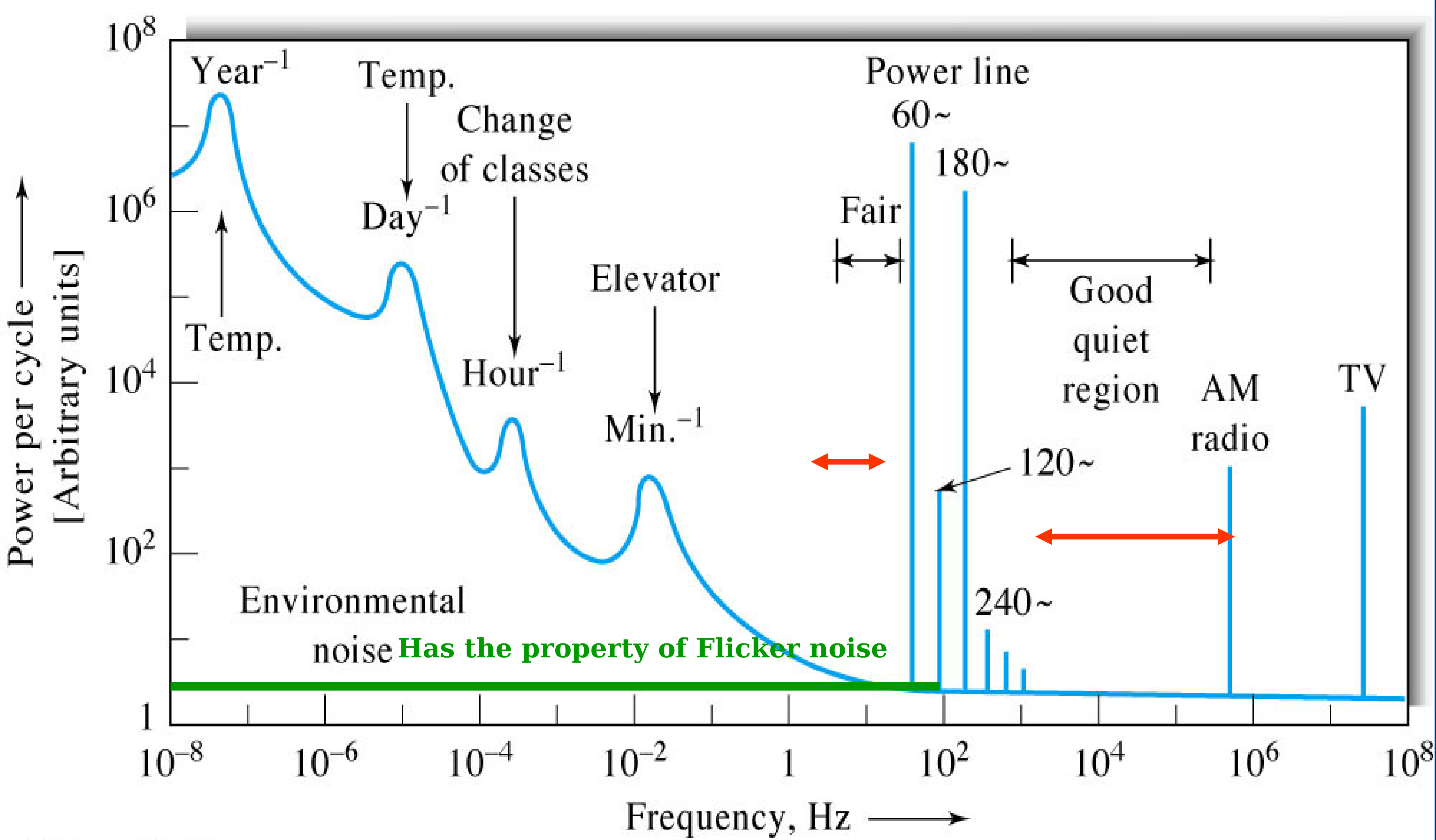
Noise types. *Brown noise*



Black noise

- Black noise is also called silent noise.
  - Noise with a $1/f^{\beta}$ spectrum, where $\beta > 2$. This formula is used to model the frequency of natural disasters.
  - Noise that has a frequency spectrum of predominantly zero power level over all frequencies except for a few narrow bands or spikes.
  - Note: An example of black noise in a facsimile transmission system is the spectrum that might be obtained when scanning a black area in which there are a few random white spots. Thus, in the time domain, a few random pulses occur while scanning

NOISE SPECRUM



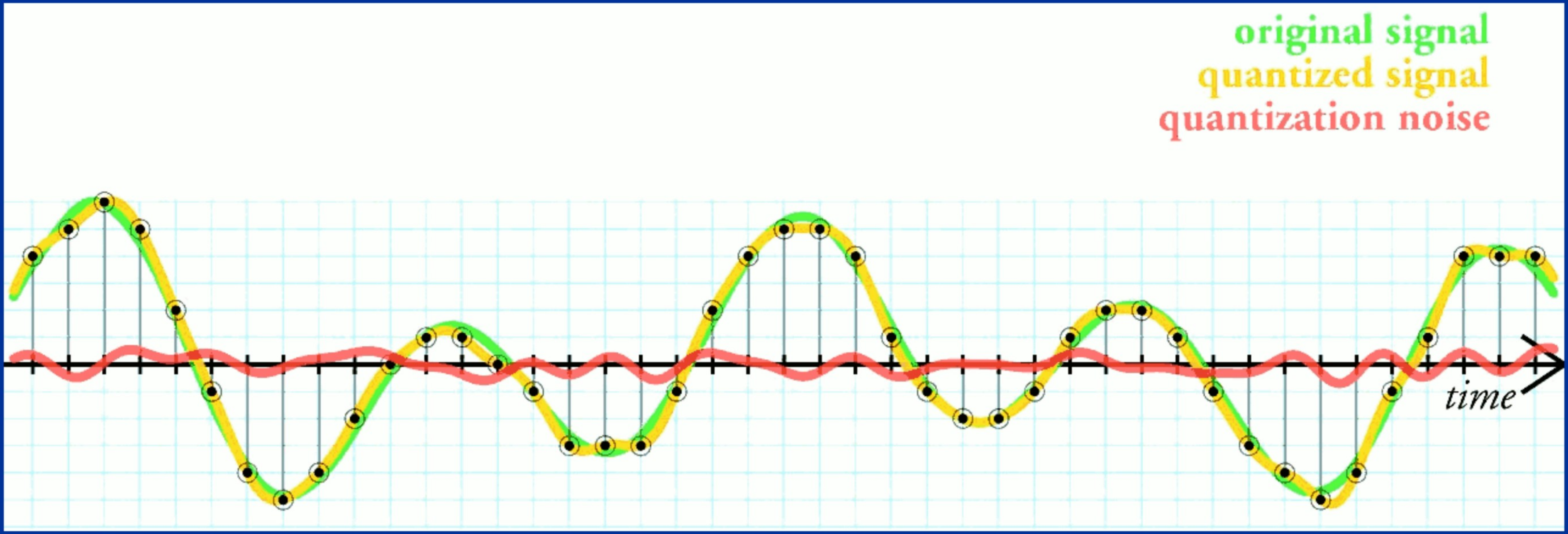
Gaussian noise

- Gaussian noise is statistical noise having a probability density function (PDF) equal to that of the normal distribution, which is also known as the Gaussian distribution.
- In other words, the values that the noise can take on are Gaussian-distributed.
- A special case is **white Gaussian noise**, in which the values at any pair of times are identically distributed and statistically independent (and hence uncorrelated).
-
- Signals can be affected by wideband Gaussian noise coming from many natural sources, such as the thermal vibrations of atoms in conductors (thermal noise or Johnson-Nyquist noise), shot noise, black body radiation from the earth and other warm objects
-
- Principal sources of Gaussian noise in digital images arise during acquisition e.g. sensor noise caused by poor illumination and/or high temperature, and/or transmission e.g. electronic circuit noise.
- In digital signal processing Gaussian noise can be reduced using a spatial filter, though when smoothing an image, an undesirable outcome may result in the blurring of fine-scaled image edges and details because they also correspond to blocked high frequencies.
- Conventional spatial filtering techniques for noise removal include: mean (convolution) filtering, median filtering and Gaussian smoothing.

Multiplicative noise

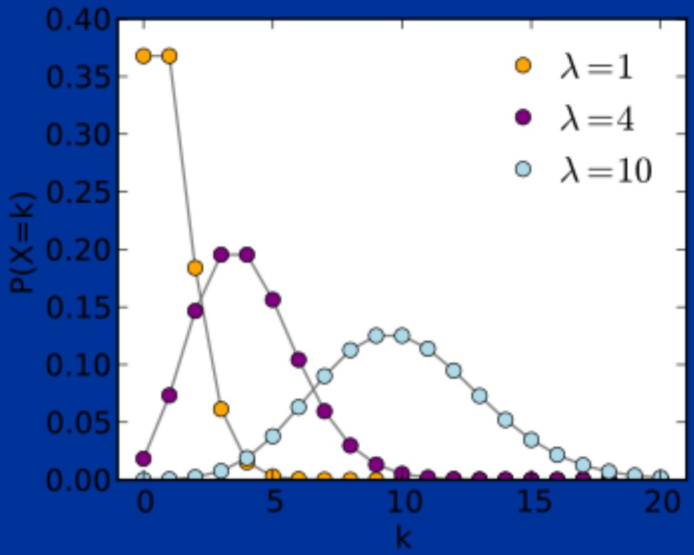
- In signal processing, the term multiplicative noise refers to an unwanted random signal that gets multiplied into some relevant signal during capture, transmission, or other processing.
-
- Examples of multiplicative noise affecting digital photographs are:
  - proper shadows due to undulations on the surface of the imaged objects,
  - shadows cast by complex objects like foliage and Venetian blinds,
  - dark spots caused by dust in the lens or image sensor,
  - variations in the gain of individual elements of the image sensor array.

Quantization error



Poisson noise

- Shot noise or Poisson noise is a type of electronic noise which can be modeled by a Poisson process. In electronics shot noise originates from the discrete nature of electric charge. Shot noise also occurs in photon counting in optical devices, where shot noise is associated with the particle nature of light.
- For large numbers, the Poisson distribution approaches a normal distribution about its mean, and the elementary events (photons, electrons, etc.) are no longer individually observed, typically making shot noise in actual observations indistinguishable from true Gaussian noise.
- Since the standard deviation of shot noise is equal to the square root of the average number of events N, the signal-to-noise ratio is given by:
-
- $SNR = N/\sqrt{N} = \sqrt{N}$
-
- Thus when N is very large, the signal-to-noise ratio is very large as well, and any relative fluctuations in N due to other sources are more likely to dominate over shot noise.
- However when the other noise source is at a fixed level, such as thermal noise, or grows slower than N, increasing N (the DC current or light level, etc.) can lead to dominance of shot noise.



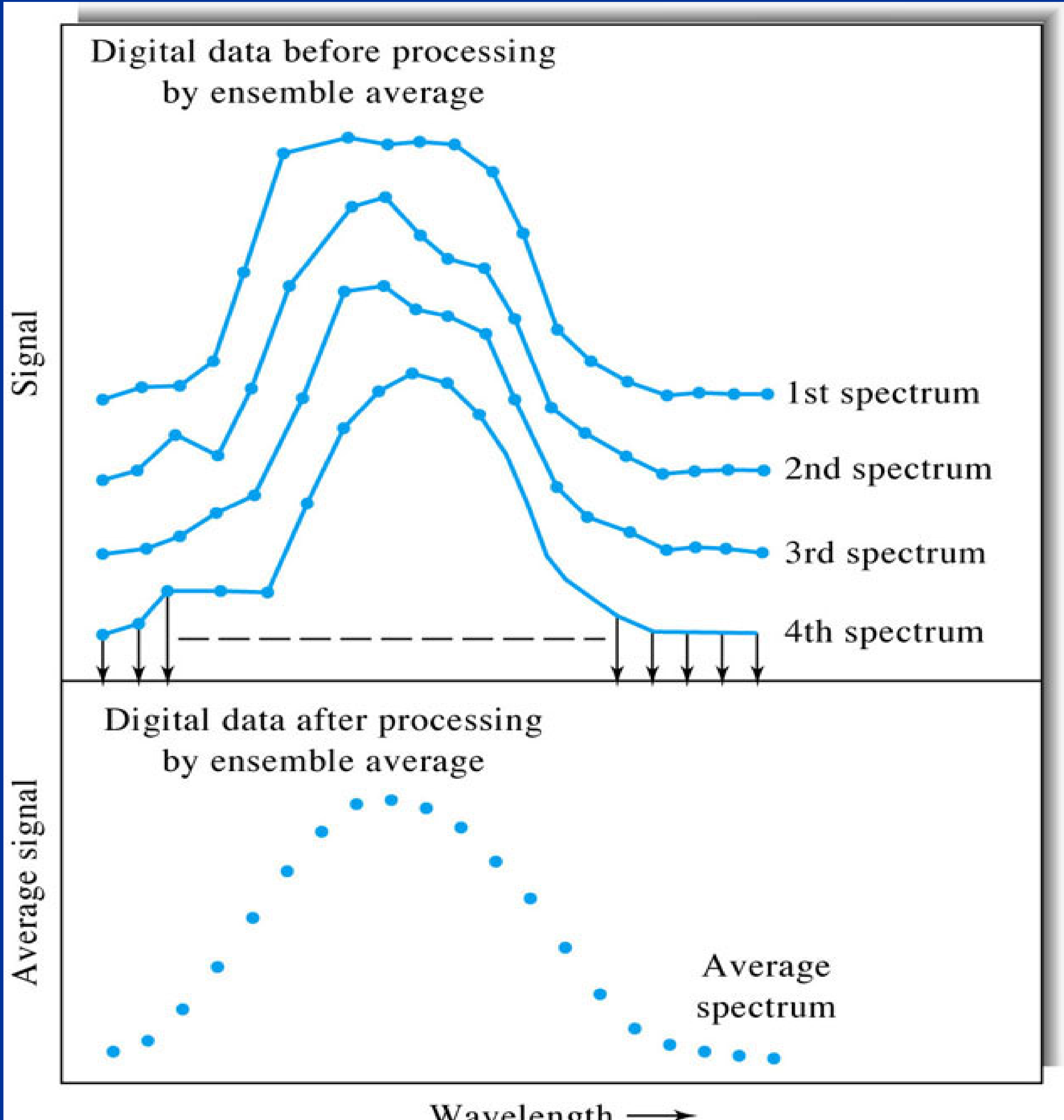
The number of photons that are collected by a given detector varies, and follows a Poisson distribution, depicted here for averages of 1, 4, and 10.

Phase noise

- Random time shifts in a signal
- Consider the following noise-free signal:
- $v(t) = A\cos(2\pi f_0 t).$
-
- Phase noise is added to this signal by adding a stochastic process represented by φ to the signal as follows:
- $v(t) = A\cos(2\pi f_0 t + \varphi(t)).$

Signal Averaging (one way of controlling noise)

- Ensemble Averaging
  - Collect multiple signals over the same time or wavelength (for example) domain
  - EASILY DONE WITH COMPUTERS!
- Calculate the mean signal at each point in the domain
- Re-plot the averaged signal
- Since noise is random (some +/ some -), this helps reduce the overall noise by cancellation!


Digital data before processing
by ensemble average
Signal
1st spectrum
2nd spectrum
3rd spectrum
4th spectrum
Digital data after processing
by ensemble average
Average signal
Average
spectrum
Wavelength

S_i, individual measurements of the signal including noise

If we sum n measurements to obtain the ensemble average, the signal S_i adds for each repetition.

The total signal S_n is given by

$$S_n = \sum_{i=1}^{n} S_i = n S_i$$

For the noise, the variance is additive (note STD is not additive). The total Variance is

$$\sigma_n^2 = \sum_{i=1}^{n} \sigma_i^2 = n \sigma_i^2$$

The standard deviation, or the total rms noise, is

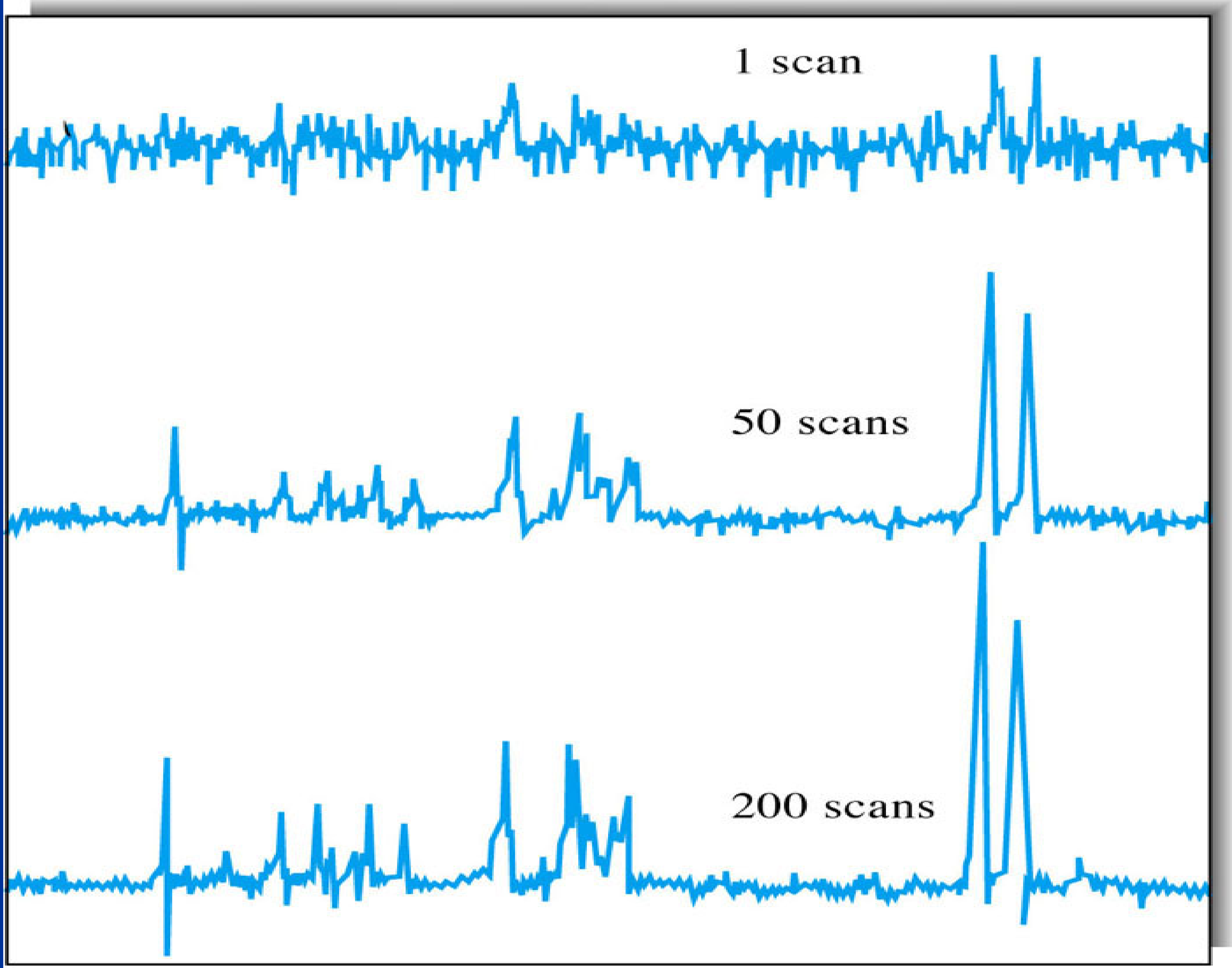
$$\sigma_n = N_n = \sqrt{n}\, \sigma_i = \sqrt{n}\, N_i$$

The S/N after n repetitions $(S/N)_n$ is then,

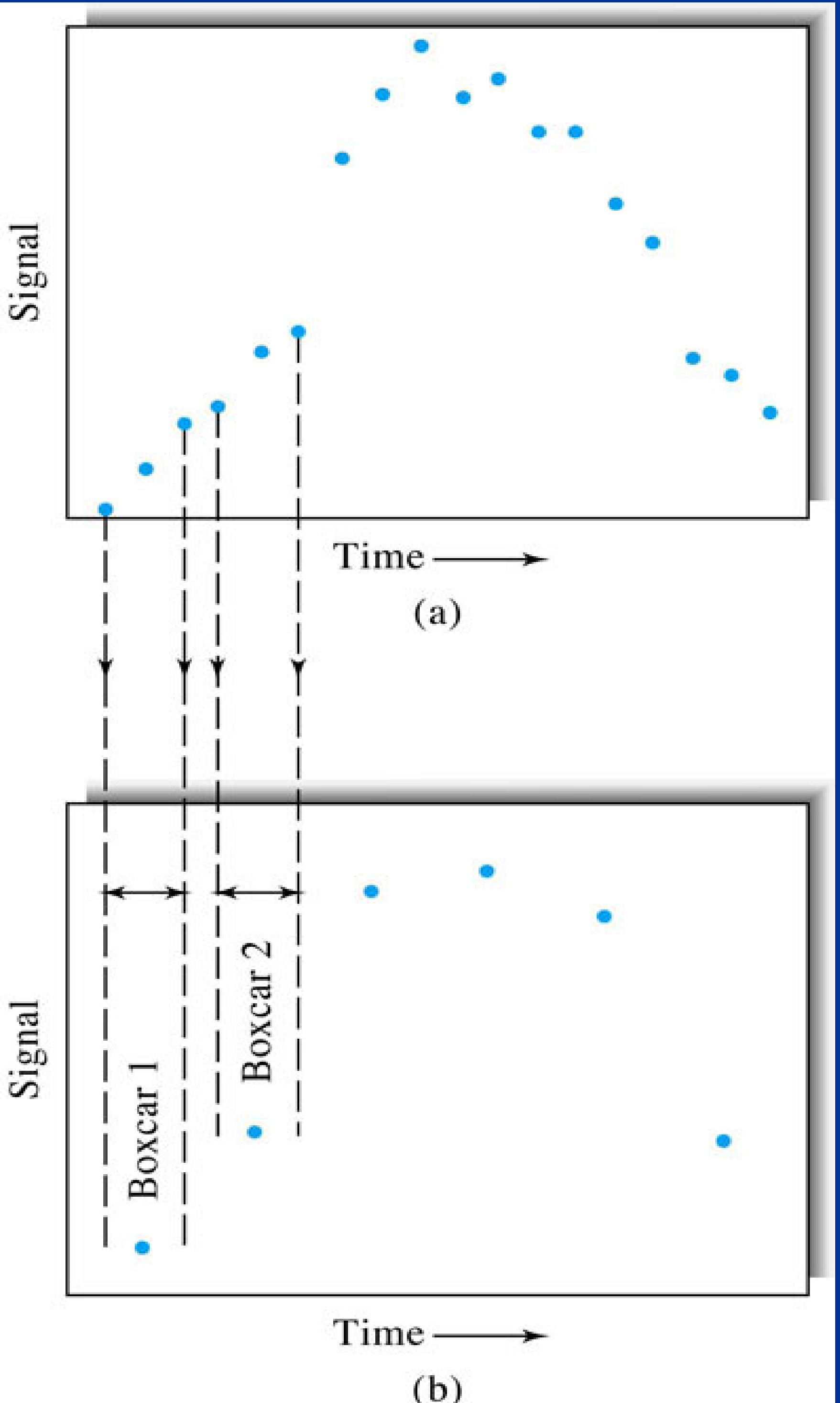
$$\left(\frac{S}{N}\right)_n = \frac{n S_i}{\sqrt{n} N_i}$$

$$\left(\frac{S}{N}\right)_n = \sqrt{n}\left(\frac{S}{N}\right)_i$$

S/N is good → KEEP ADDING!



- **Boxcar Averaging**
  - **Take an average of 2 or more signals in some domain**
  - **Plot these points as the average signal in the same domain**
  - **Can be done with just one set of data**
  - **You lose some detail in the overall signal**

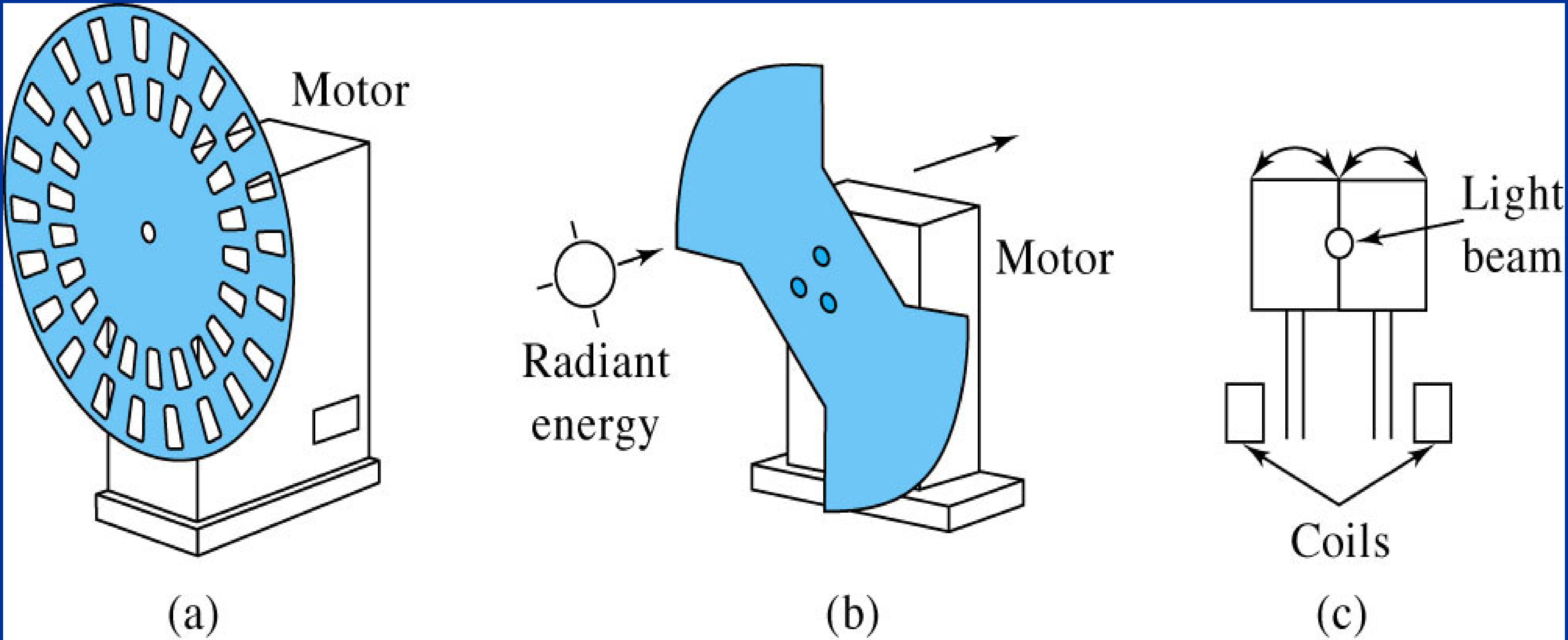


Signal
Time
(a)
Signal
Boxcar 1
Boxcar 2
Time
(b)

Hardware device for noise reduction

- Grounding and shielding
- Differences and instrumentation amplifiers
- Analog filtering
- Modulation

Hardware device for noise reduction



Hardware device for noise reduction