Hilbert-Huang Transform(HHT)

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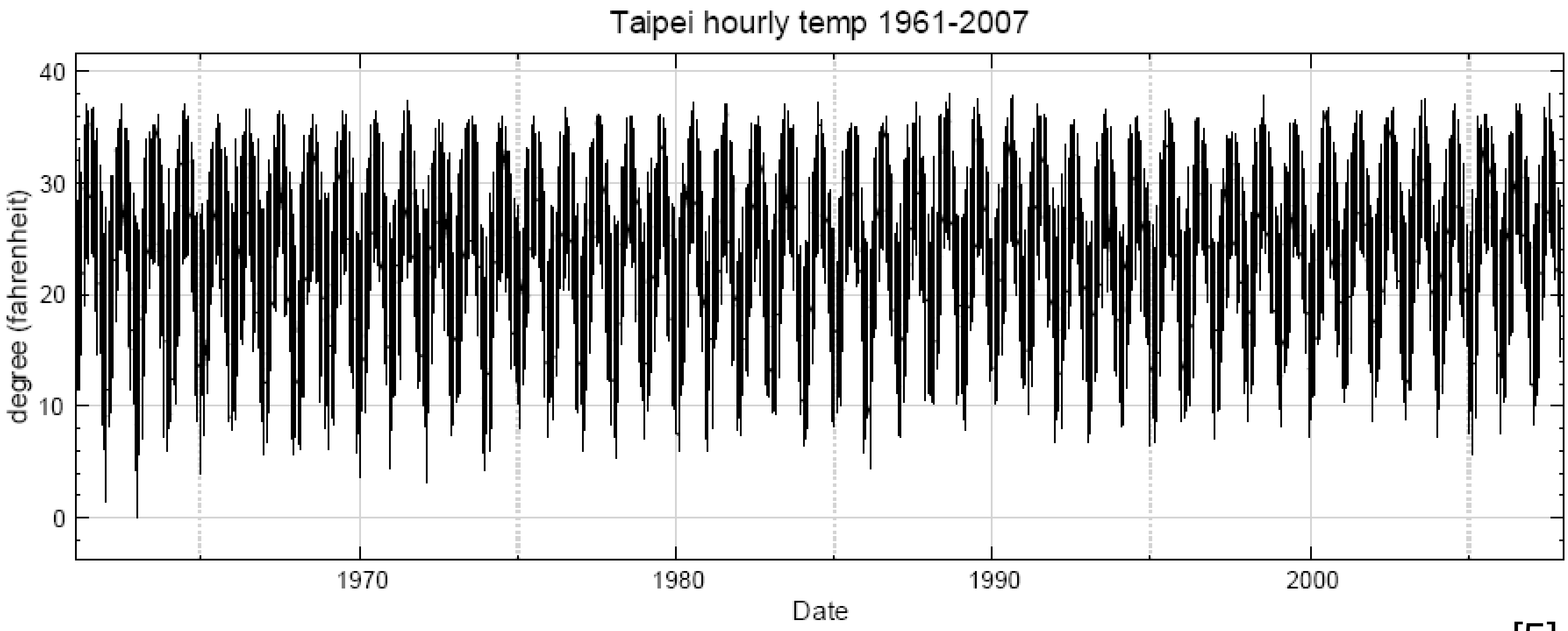
Motivation

- To deal with nonlinear and non-stationary signal
- To get Instantaneous frequency(IF)



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Hilbert Transform

- The Hilbert transform can be thought of as the convolution of s(t) with the function h(t) = 1/(πt)

$$\hat{s}(t) = s(t) * \frac{1}{\pi t}$$

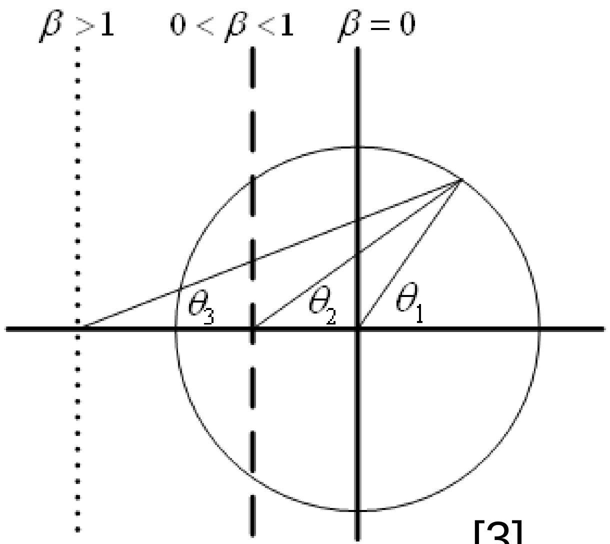
- Derive the analytic representation of a signal

$$z(t) = s(t) + j\hat{s}(t) = m(t) \cdot e^{j\theta(t)}$$

$$\text{Instantaneous Frequency : f(t)} = \frac{d}{dt}\theta(t)$$

Instantaneous Frequency(IF)

- $s(t) = \beta + \cos(t)$
  - (1) $\beta = 0$: IF is the constant
  - (2) $0 < \beta < 1$: IF has been oscillating
  - (3) $\beta > 1$: IF has been negative



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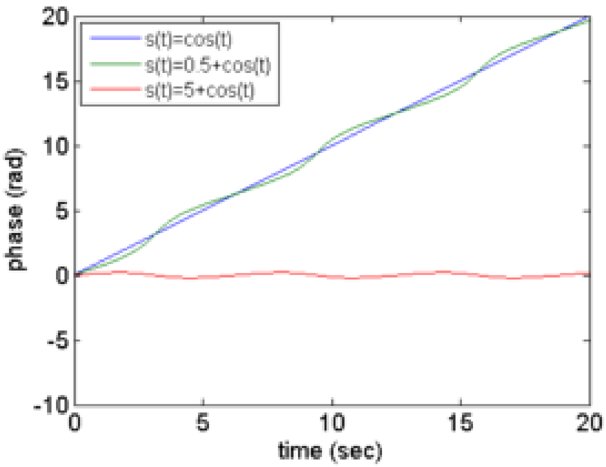

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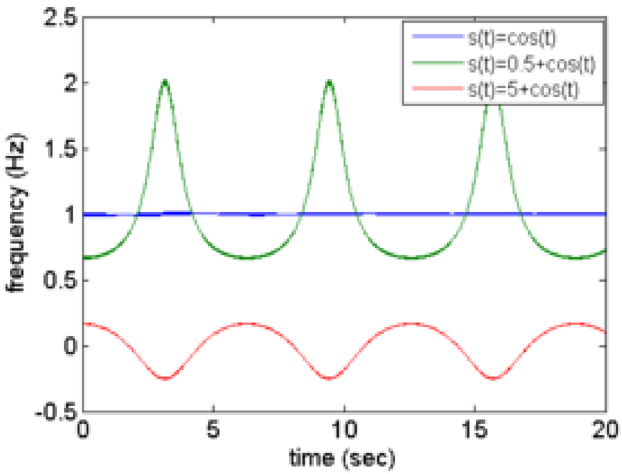
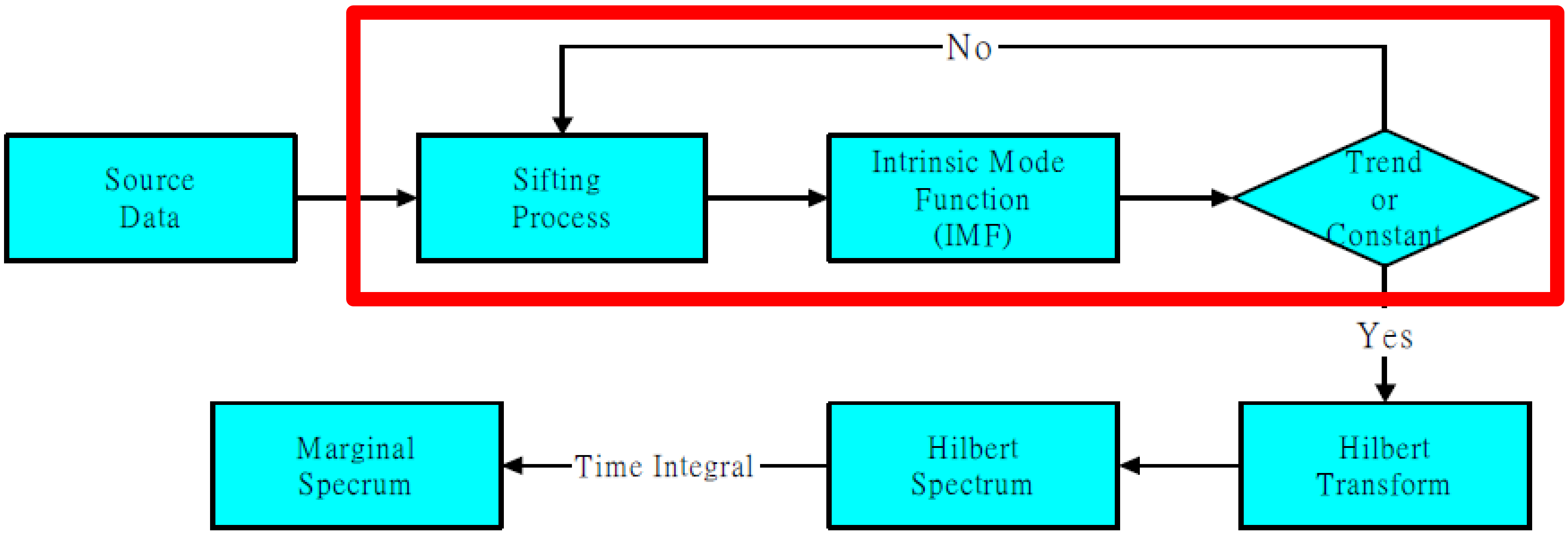


[3]

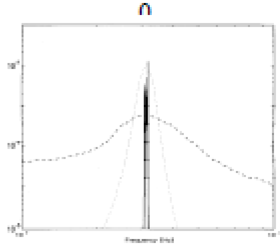
Flow Chart

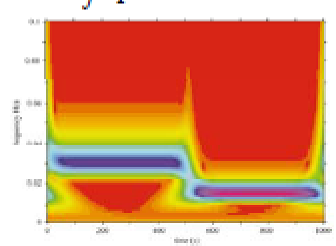


$$h(\omega) = \int_0^T H(\omega, t)\,dt$$

$$X(t) = \sum_{j=1}^{n} a_j(t)\, e^{i2\pi \int f_j(t)dt}$$

$$Y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{X(t')}{t - t'}\, dt'$$





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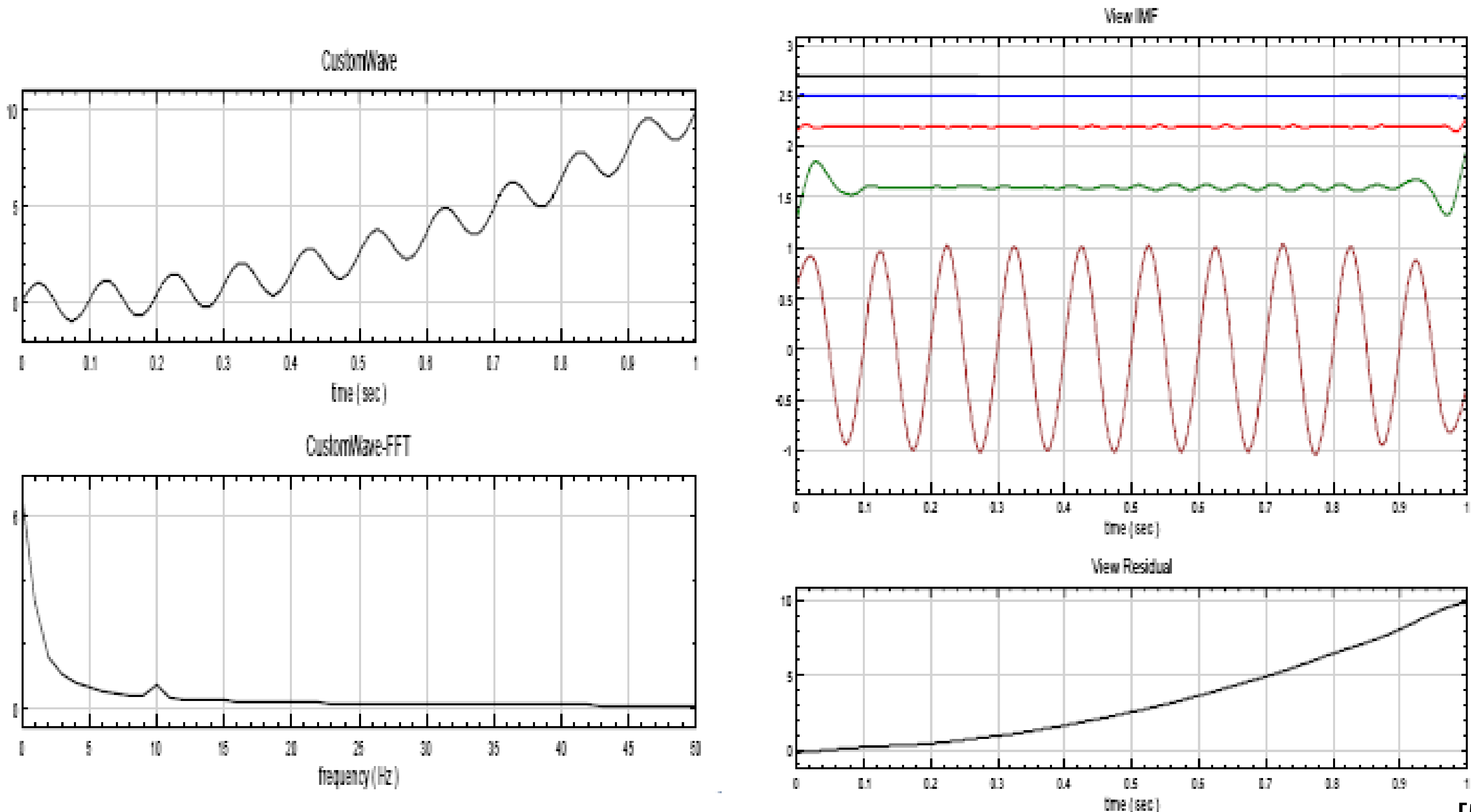
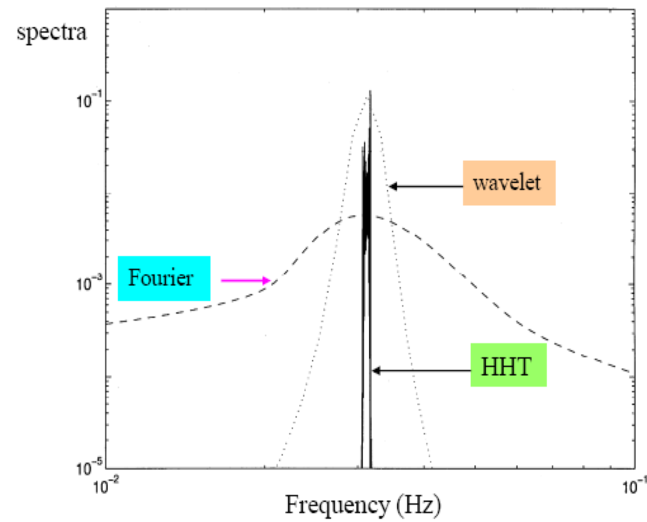
Example

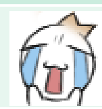
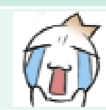
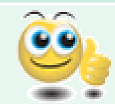
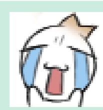
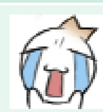
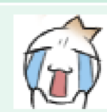
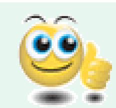

CustomWave
time (sec)
CustomWave-FFT
frequency (Hz)
View IMF
time (sec)
View Residual
time (sec)

[5]

Time–Frequency Analysis

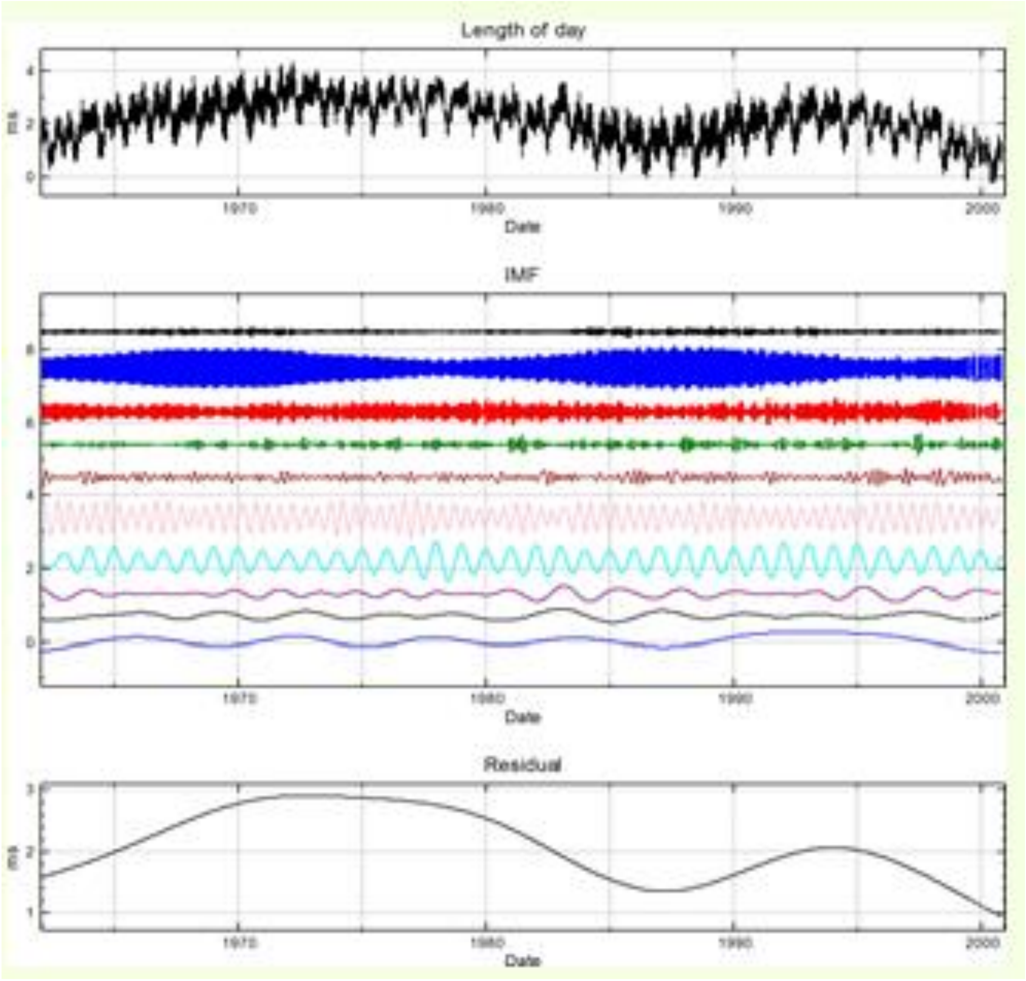
- Fast Fourier Transform (FFT)
- Wavelet Transform
- Hilbert-Huang Transform (HHT)



	FFT	Wavelet	HHT
Basis	*a priori*	*a priori*	Adaptive
Nonlinear			
Non-stationary			
Feature Extraction			

Geoscience

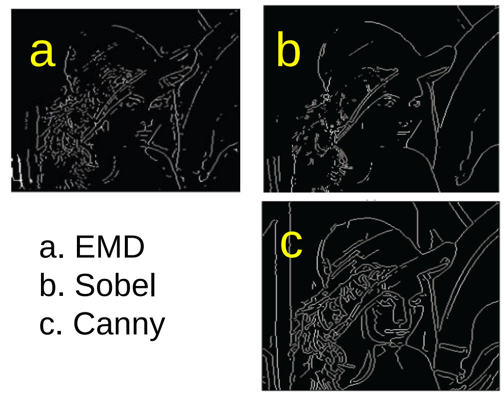
- Length of day



[5]

Image Processing

- Edge detection [10]
- Image denoise [11]
- Image fusion [12]



a. EMD
b. Sobel
c. Canny

image	wavelet	our method
Lena: 52.9715	59.8365	61.1207
Barb: 53.1116	60.3938	61.0224

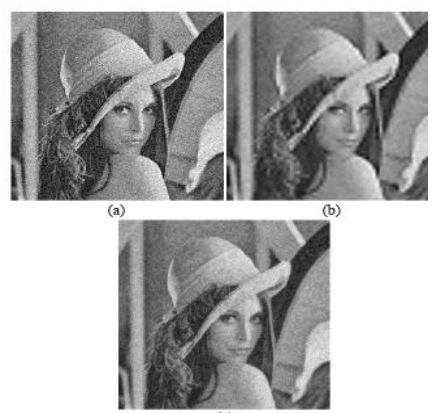


Figure 3. (a) image with random noise(PSNR=52.9715); (b) wave method; (c)the proposed method

image	wavelet	our method
Lena: 31.3933	52.5294	53.1010
Barb: 31.4267	53.0473	53.7436.

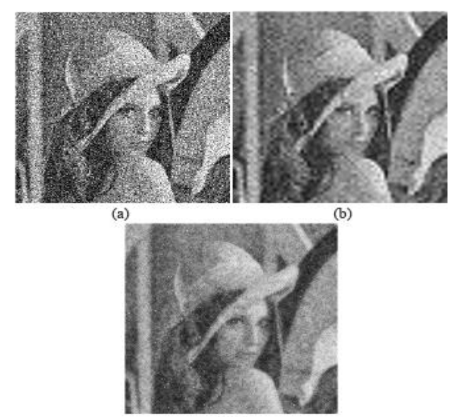


Figure 4. (a) image with random noise(PSNR=31.3933); (b) wavelet method; (c)the proposed method

Problems of HHT

- P1: Stopping criterion
- P2: End effect problem
  - Hilbert Transform
  - ALIF
- P3: Mode Mixing
- P3: Speed of computing

P1: Stopping Criterion

- Standard deviation(SD)

$$SD = \sum_{t=0}^{T} \left[\frac{\left|(h_{(k-1)}(t) - h_k(t))\right|^2}{h^2_{(k-1)}(t)} \right]$$ [1]

  - $SD \leq 0.2 \sim 0.3$

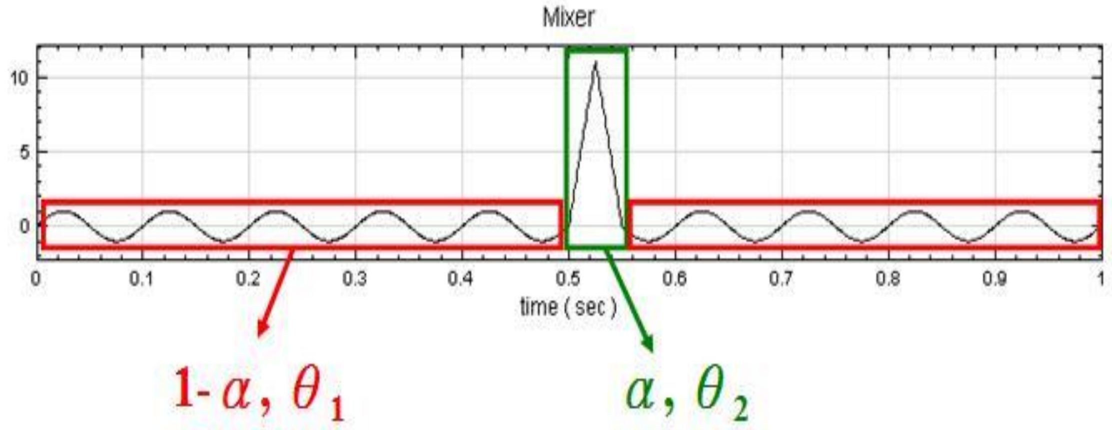
- S number criterion

$$\text{Orthogonal Index(OI)} = \sum_{t=0}^{T} \frac{C_f C_g}{C_f^2 + C_g^2}$$ [2]

  - $3 \leq S \leq 5$

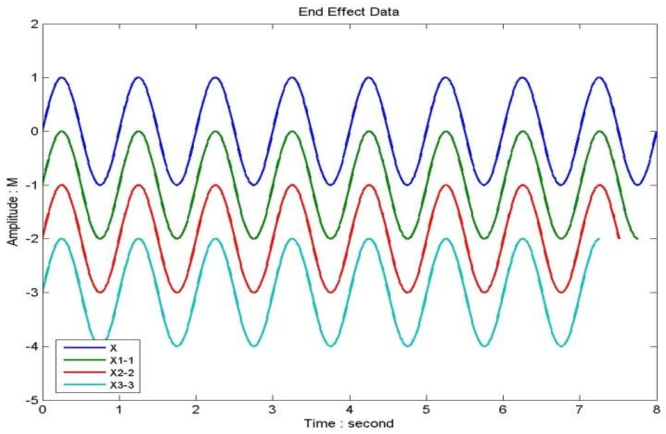
- Three parameter method(θ_1,θ_2, α) [3]
  - Mode amplitude : $a(t) = (e_{\max}(t) - e_{\min}(t))/2$
  - Evaluation function : $\sigma(t) = \left| m(t)/a(t) \right|$
  - $\sigma(t) < \theta_1$ in $(1-\alpha)$

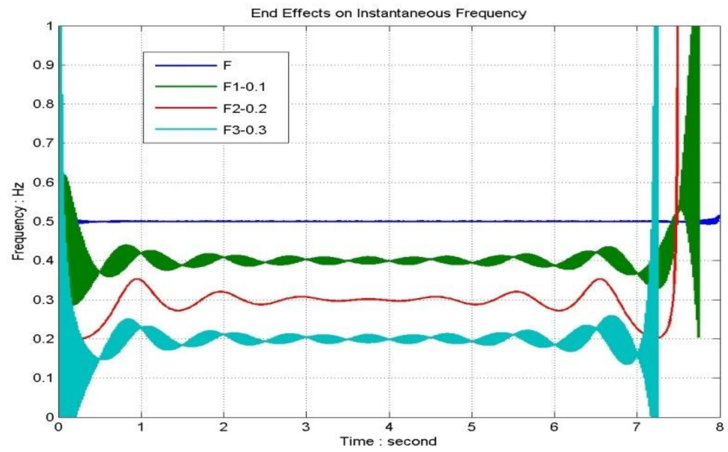
    $\sigma(t) < \theta_2$ in α
  - $\alpha \fallingdotseq 0.05$, $\theta_1 \fallingdotseq 0.05$, $\theta_2 \fallingdotseq 10\theta_1$



P2: End Effect Problem

- End effect of Hilbert Transform





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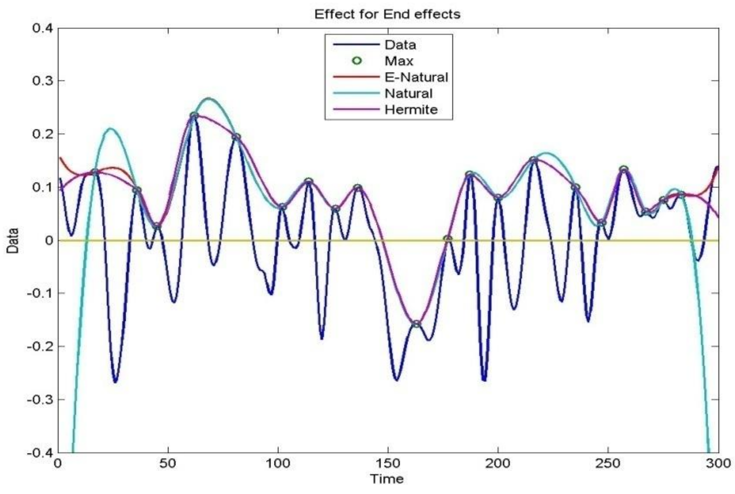
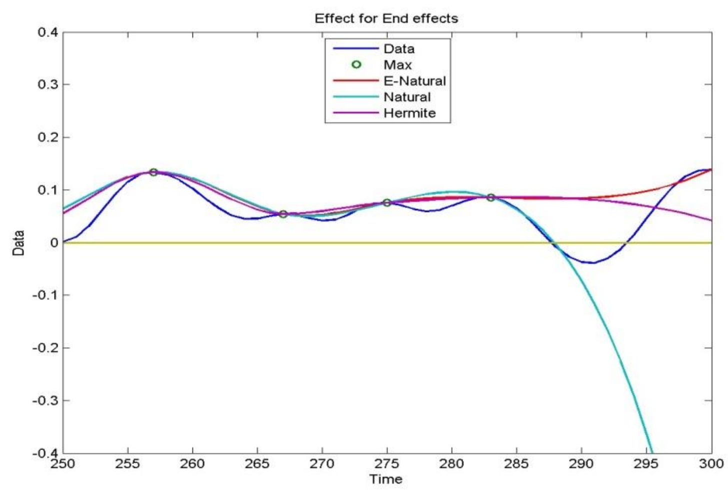
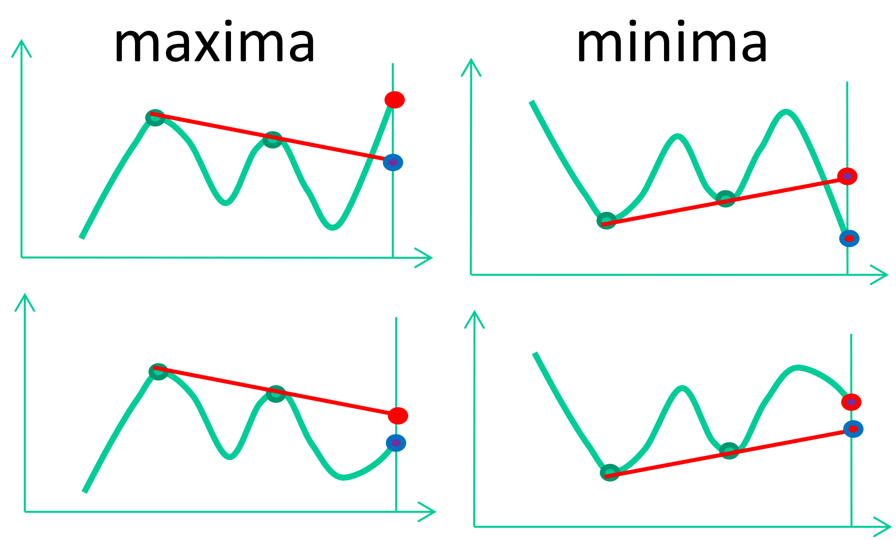
- End effect of ALIF

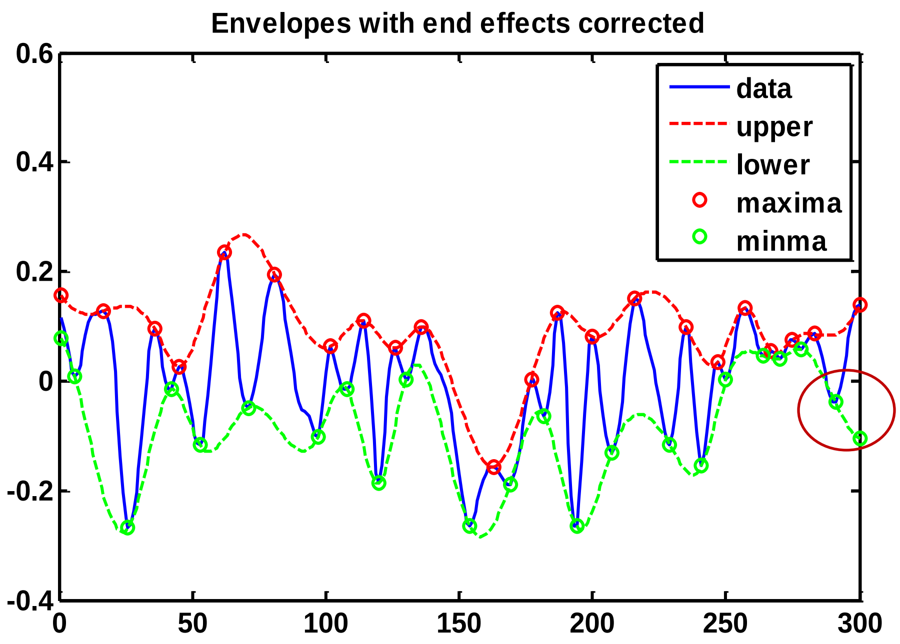




P2: Solutions for End Effects

- End effect of Hilbert Transform
  - Adding characteristics waves
- End effect of ALIF
  - Extension with linear fittings near the boundaries





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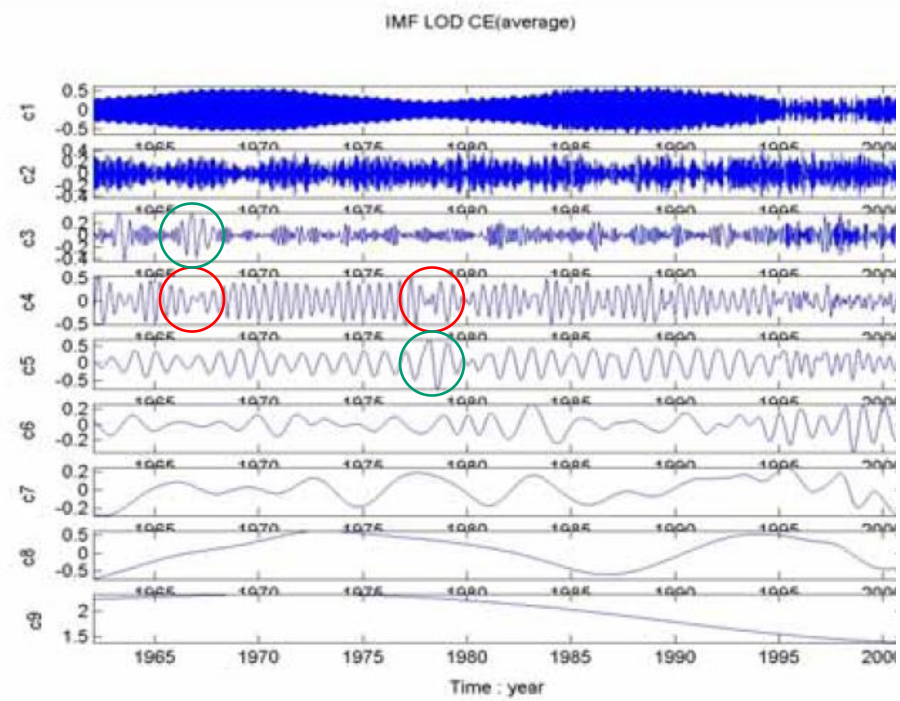
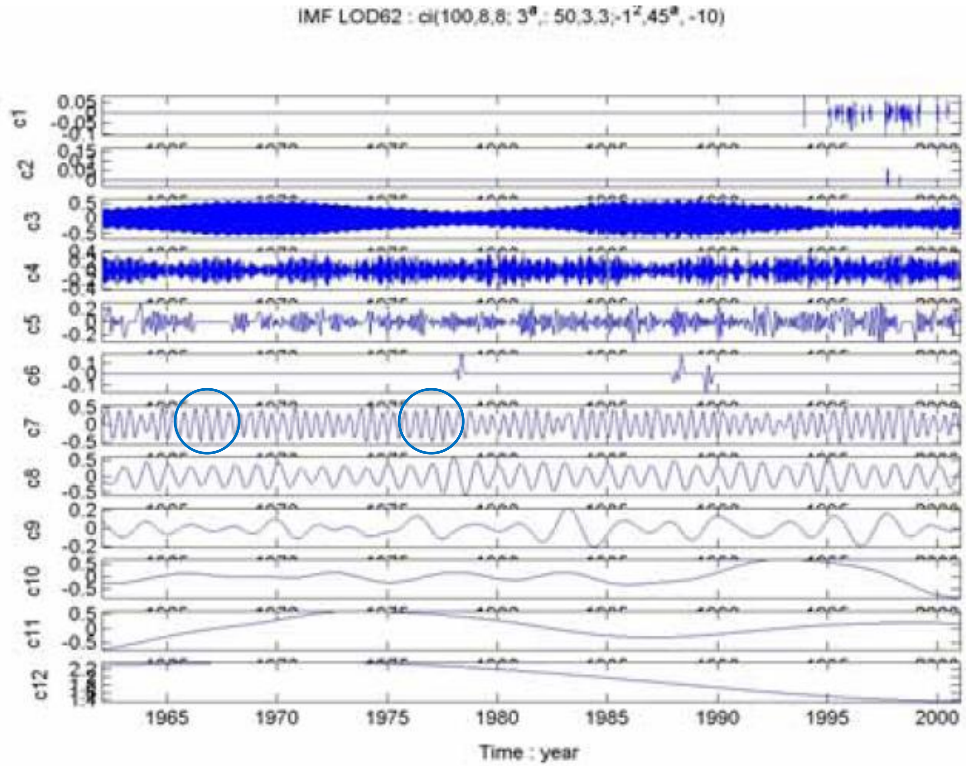
P3: Mode Mixing





❖ Post-processing of ALIF

[1]

P3: Post-Processing of ALIF

- Post-processing ALIF can get real IMFs.

EEMD

$$\mathrm{IMF}_1 = \frac{1}{m}\sum_{i=1}^{m} IMF_{i1}$$

$$\mathrm{IMF}_2 = \frac{1}{m}\sum_{i=1}^{m} IMF_{i2}$$

$$\vdots$$

$$\mathrm{IMF}_k = \frac{1}{m}\sum_{i=1}^{m} IMF_{ik}$$

Post-processing of EEMD

$$\mathrm{IMF}_1 \to \mathrm{pIMF}_1 + residual_1$$

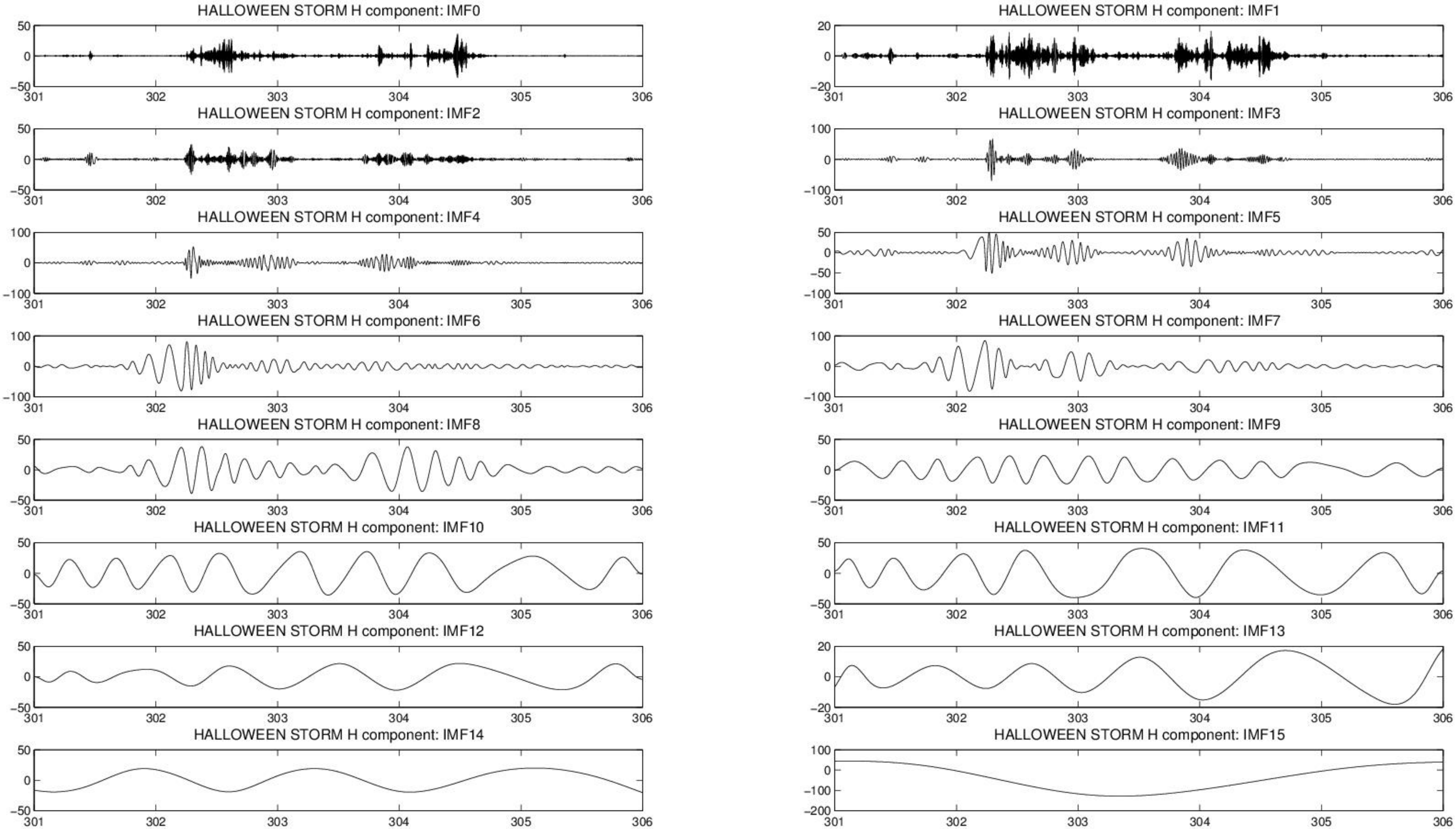
$$\mathrm{IMF}_2 + residual_1 \to \mathrm{pIMF}_2 + residual_2$$

$$\vdots$$

$$\mathrm{IMF}_k + residual_{k-1} \to \mathrm{pIMF}_k + residual_k$$

$$\Rightarrow \mathrm{X(t)} = \sum_{\mathrm{j=1}}^{\mathrm{k}} \mathrm{pIMF}_\mathrm{k} + trend$$

Example- Halloween Super Storm



ALIF – Statistical approach

What is the physical meaning of those components?

Could we sum or partially sum them? What is the way to eventually sum them?

Let's use some statistics;

We developed a test (SMT, Standardized Mean Test) that help us to sum the IMFs

We repeated the same analysis with very quiet days (SSQ; $K_p \approx$ 0/0+) in order to identify and detect the physical meaning of the partial sums of the IMFs;

We compared the results obtained at ground to those obtained for magnetospheric observations.

ALIF – SMT

The basic idea is: if a clear time scale separation exists in a data set $s(t)$, this can be divided into two different contributions[Flandrin et al., 2004]:

$$s(t) = \delta s(t) + s_0(t)$$

$s_0(t)$ represents the baseline; $\delta s(t)$ are the variations around the baseline.

IDEA: $\delta s(t)$ has the following characteristics:

1. has close to zero standardized mean (μ/σ);
2. represents the fluctuating/oscillatory contribution to the time series $s(t)$.

ALIF – SMT

- Using the orthogonality and completeness properties of EMD, we define $\delta s(t)$ as the reconstruction of a subset of the first k empirical modes (IMF – c_j) which satisfies the previous two properties, that is:

$$\delta s(t) = \sum_{j=1}^{k} c_j$$

k is the last IMF (c_j) for which $\delta s(t)$ has $\mu / \sigma \approx 0$.

ALIF – Halloween S

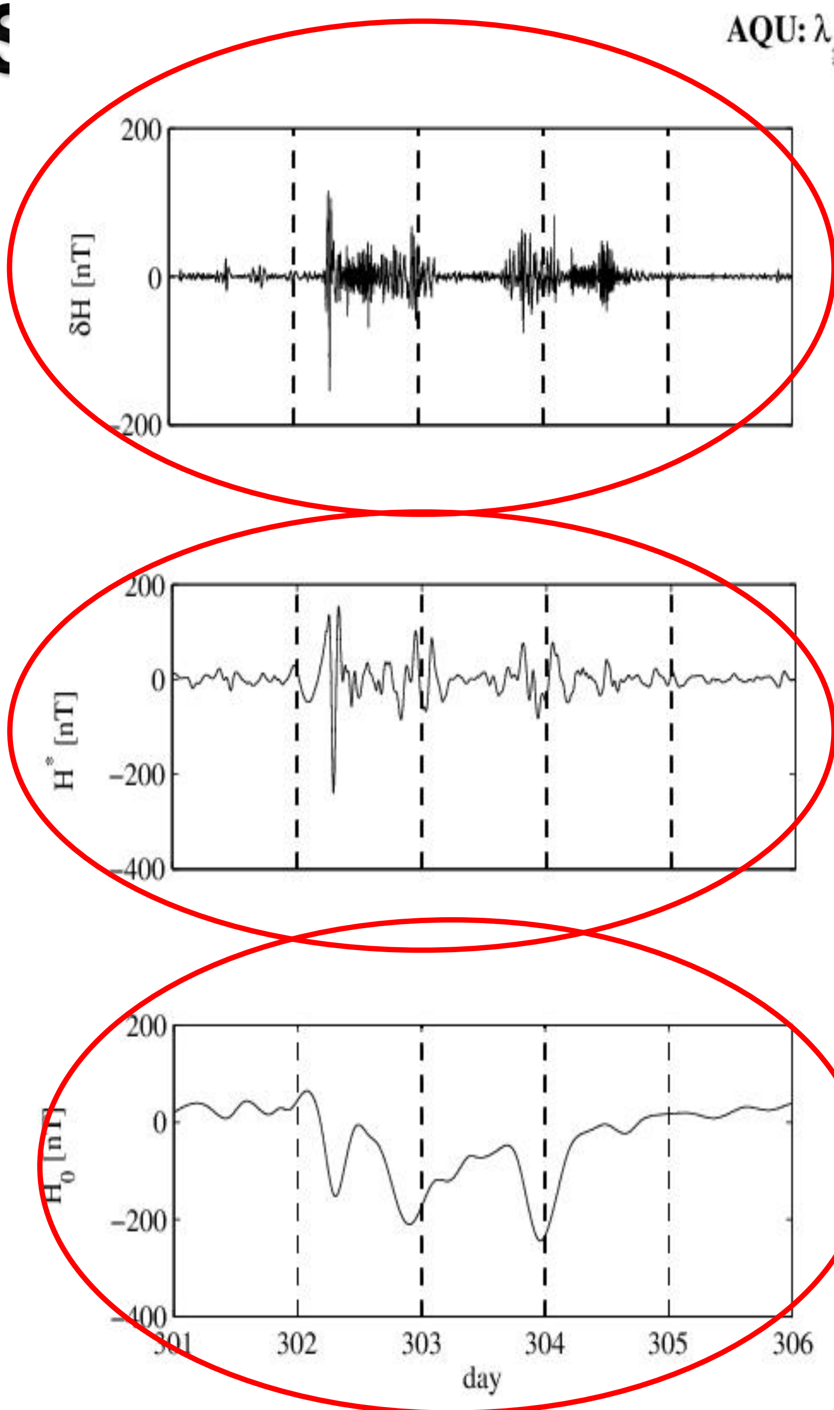
Repeting the same analysis for the HGS:

The SMT applied to the EMD gives 3 contribution at AQU e 2 contributions at both GOES.

H*(t) gives a representation of the ionospheric contribution. It lacks in the magnetospheric recontruction and presents characteristics periods between 6-24 h.

$H_0(t)$ during a GS shows the typical behaviour of a storm: a SI followed by a Main Phase and a Recovery Phase.

δH are the high frequency fluctuations, related to the turbulent character of the SW .



ALIF – MSA

For each IMF we evaluate the PDF ($p(IMF)$), probability distribution function, with a defined binning (n_{bin}=sqrt(t_c), Flandrin et al., 2004; Materassi et al.,2009; Piersanti et al., 2016).

The STD, the excess of Kurtosis (K_{ex}=K-3) e the Skewness was evaluated;

The relative (ε_{rel}) e the Shannon Information (I) was evaluated:

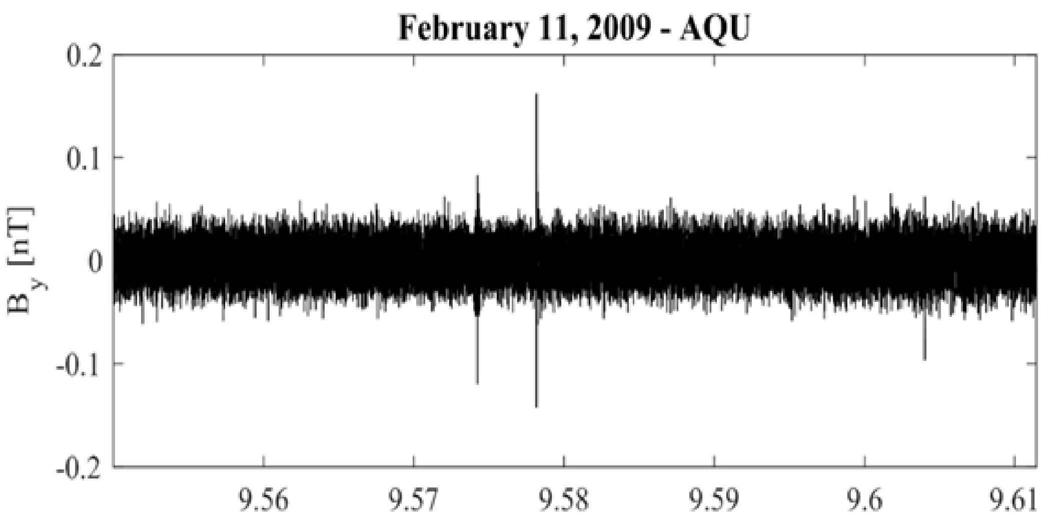
$$\epsilon_{rel} = \frac{\int_S |IMF_k(t)|^2 dt}{\int_S |s(t)|^2 dt}$$

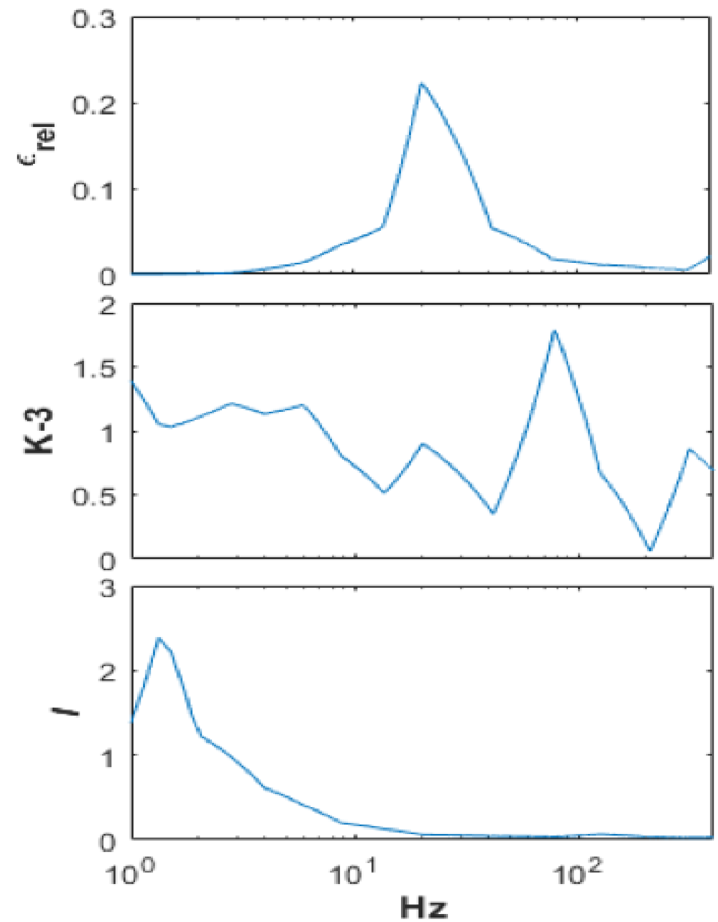
$$I = -\sum_{\{IMF_k\}} p(IMF_k) \cdot log_2 p(IMF_k)$$

ALIF – MSA – Example DEMETER EFD

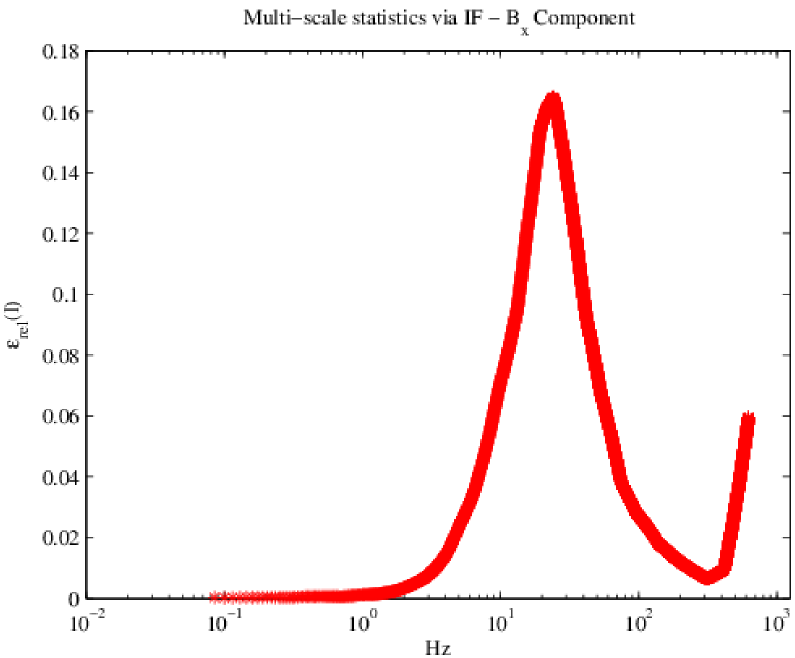
Since:

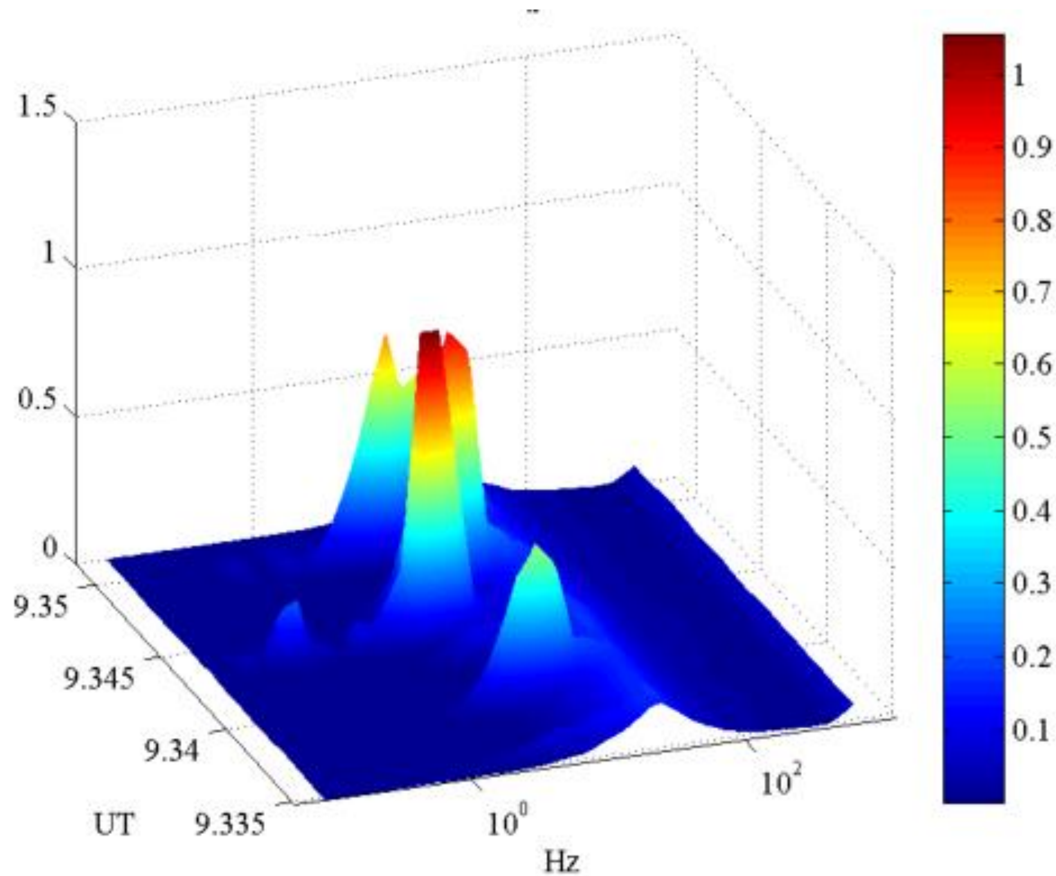
1. Kex measure how abundant are the rare fluctuation at different frequencies;
2. I measure the «casual degree» for each scale (for each frequency.





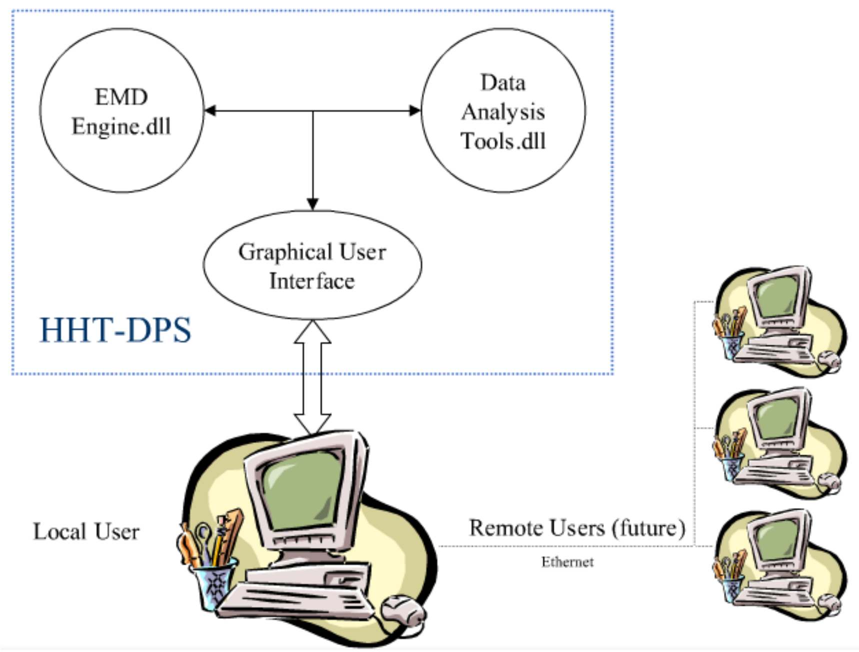
3D RELATIVE ENERGY





P4: Speed of Computing

- The processing time of HHT is dependent on complexity of the data and criterions of the algorithm
- HHT data processing system(HHT-DPS)
- Implementation of HHT based on DSP [13]



Conclusion

- The definition of an IMF guarantees a well-behaved Hilbert transform of the IMF
- IMF represents intrinsic signature of physics behind the data
- Although there are still many problems in HHT,HHT has lots of applications in all aspects

Thank you