

On Empirical Mode Decomposition and its Algorithms

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outline

- Empirical Mode Decomposition (EMD)
basics
- examples
- algorithmic issues
- elements of performance evaluation
- perspectives

basic idea

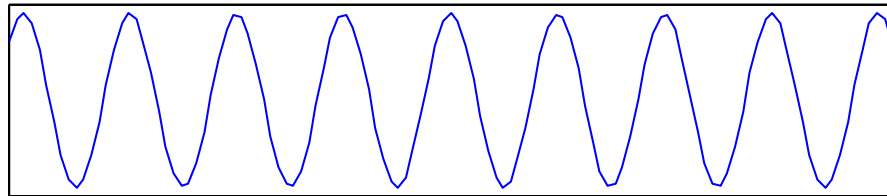
- « multimodal signal = fast oscillations on the top of slower oscillations »
- Empirical Mode Decomposition (Huang)
 - identify *locally* the fastest oscillation
 - *subtract* to the signal and *iterate* on the residual
 - *data-driven* method, locally adaptive and multiscale

Huang's algorithm

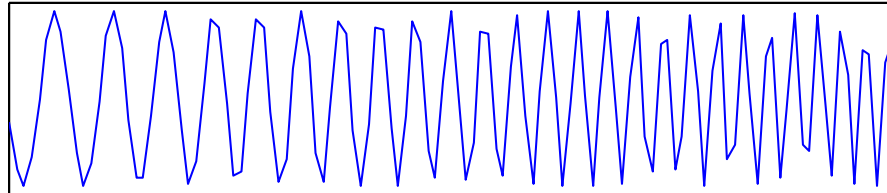
- compute lower and upper envelopes from interpolations between extrema
 - subtract mean envelope from signal
 - iterate until mean envelope = 0 and $\#\{\text{extrema}\} = \#\{\text{zero-crossings}\} \pm 1$
- subtract the obtained *Intrinsic Mode Function* (IMF) from signal and iterate on residual

how EMD works

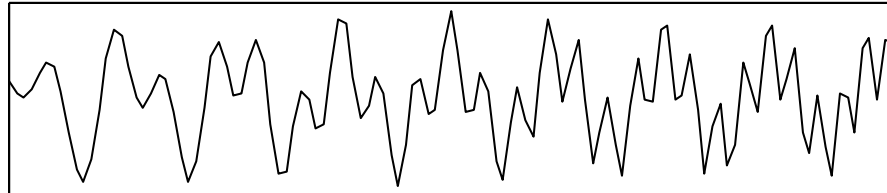
tone



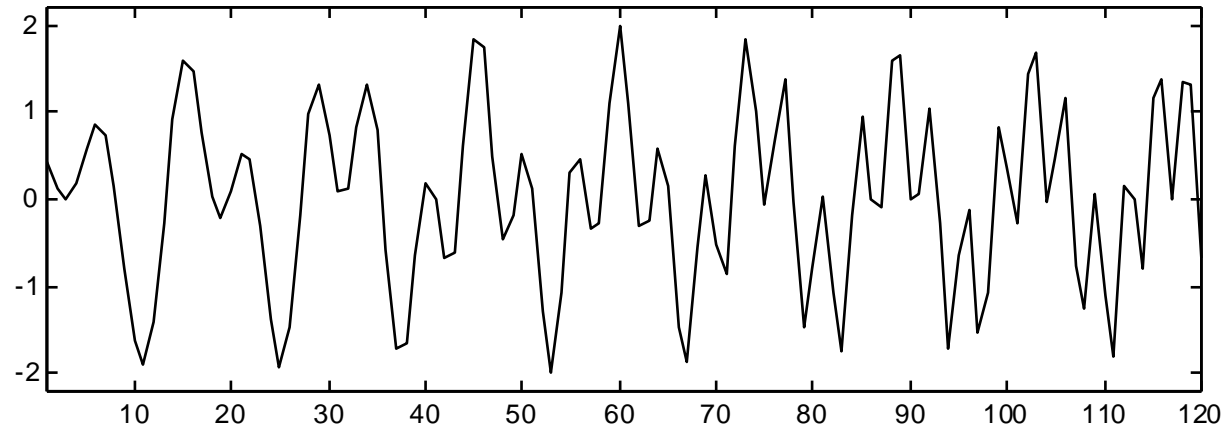
chirp

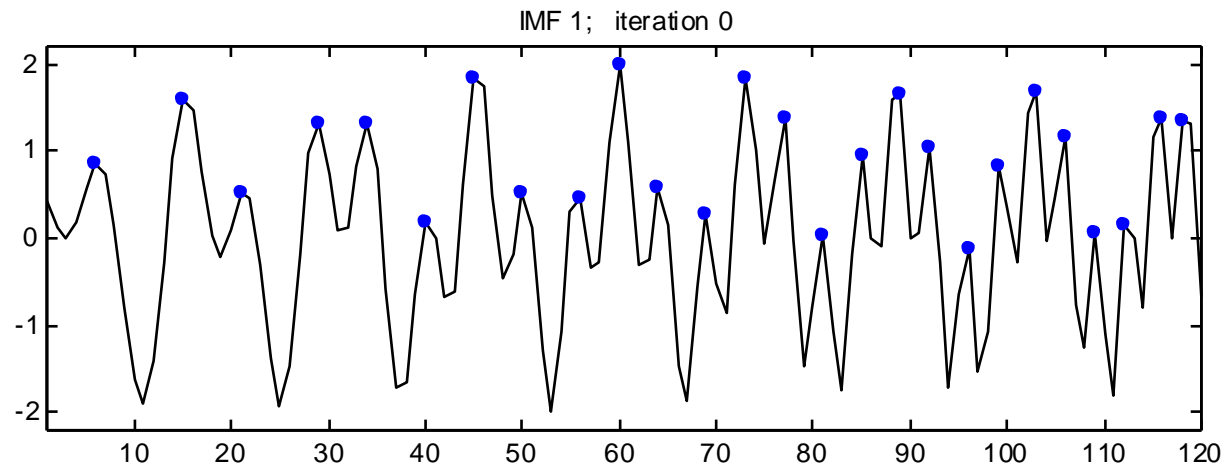


tone + chirp

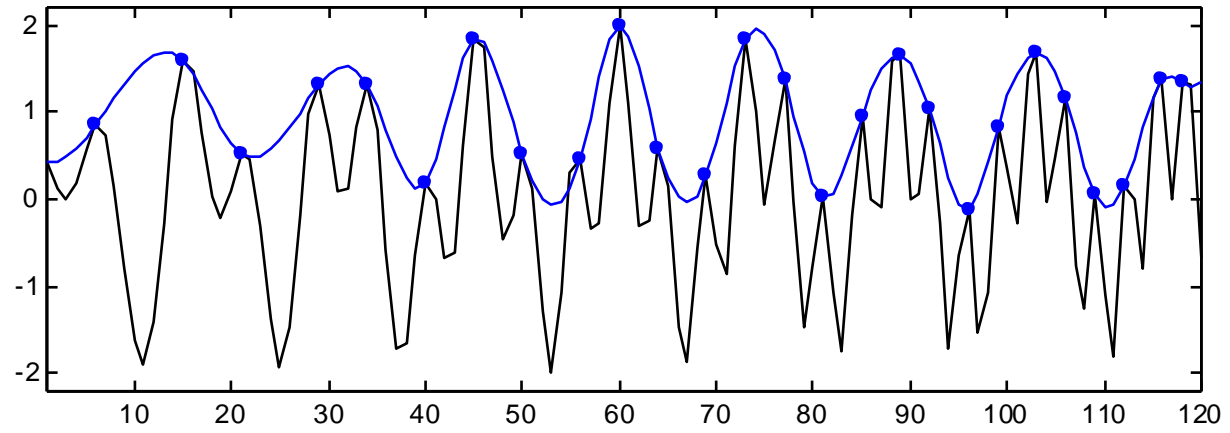


IMF 1; iteration 0

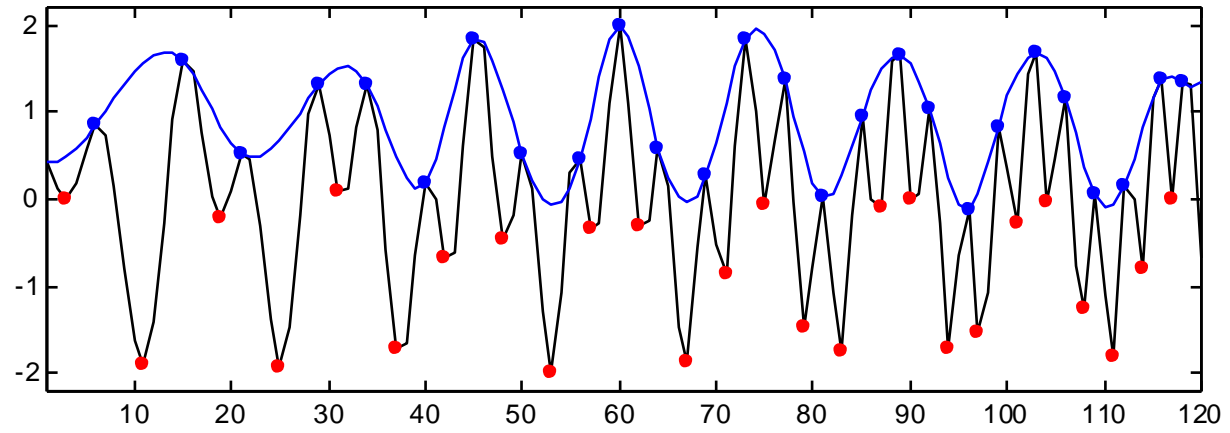




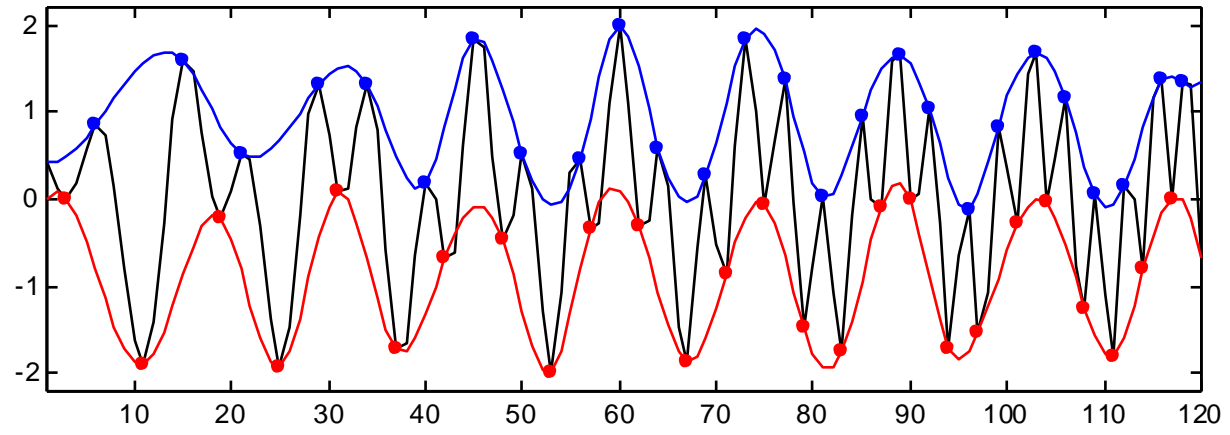
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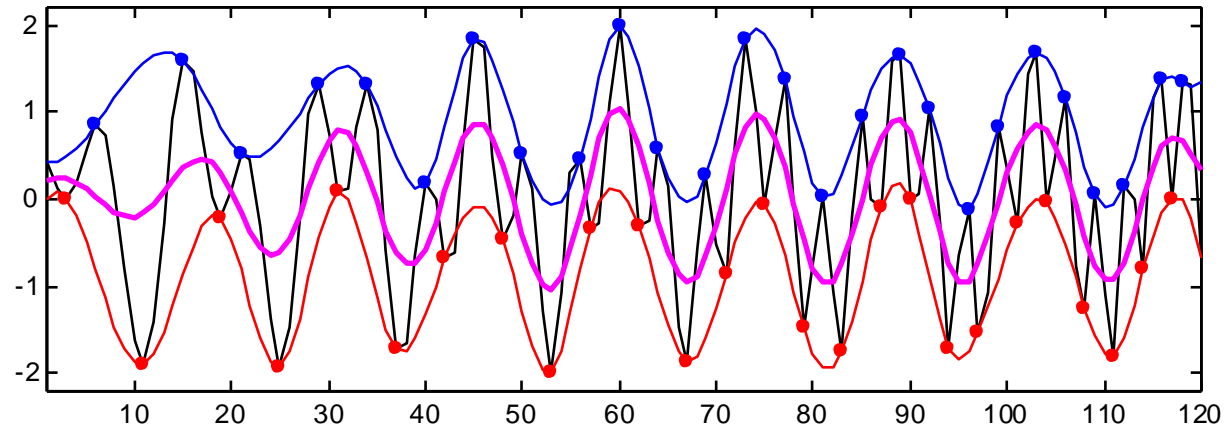
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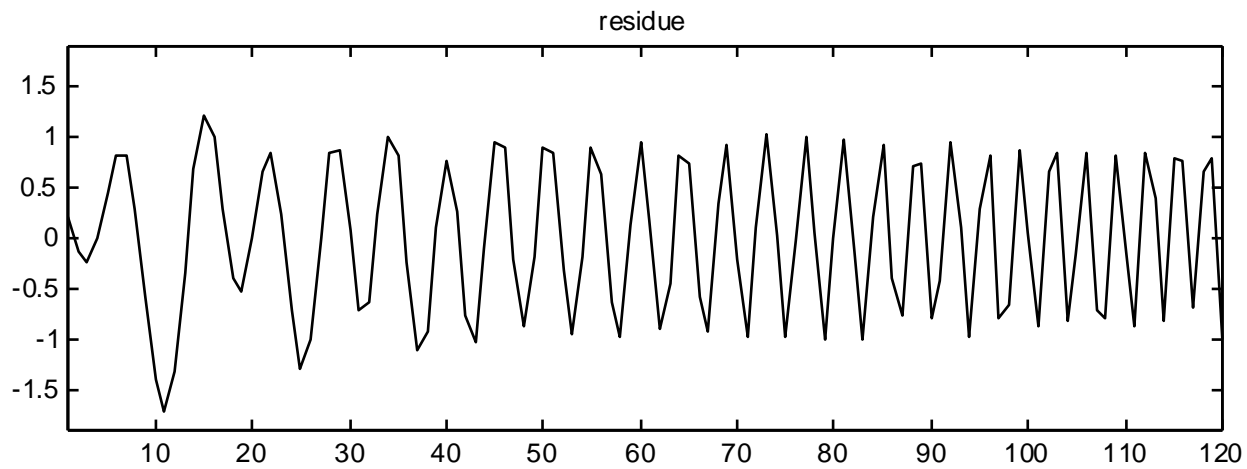
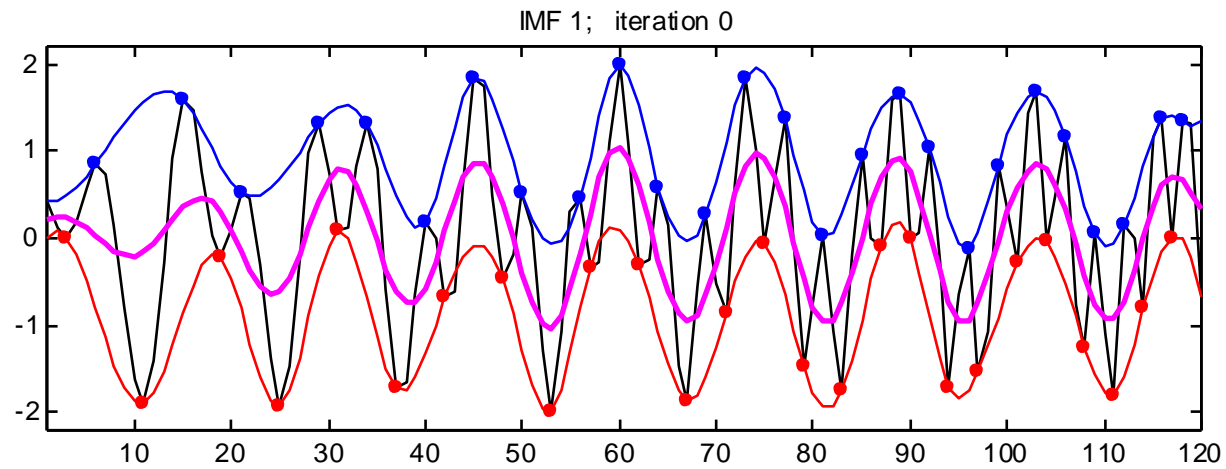


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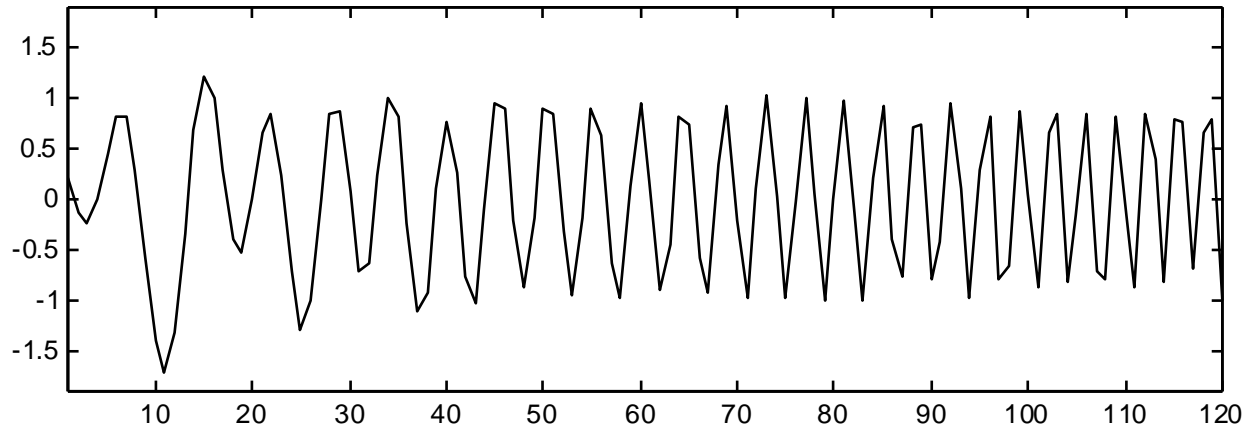


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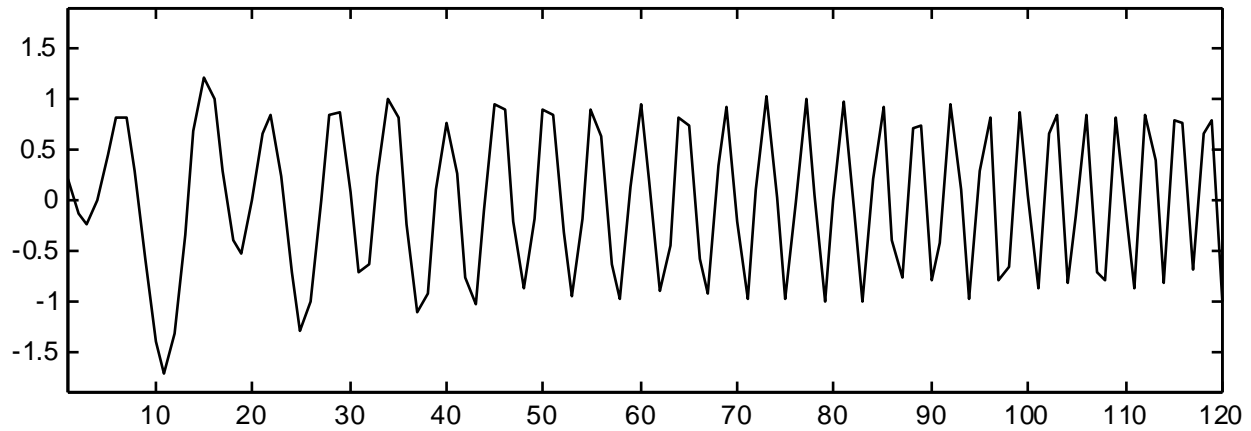




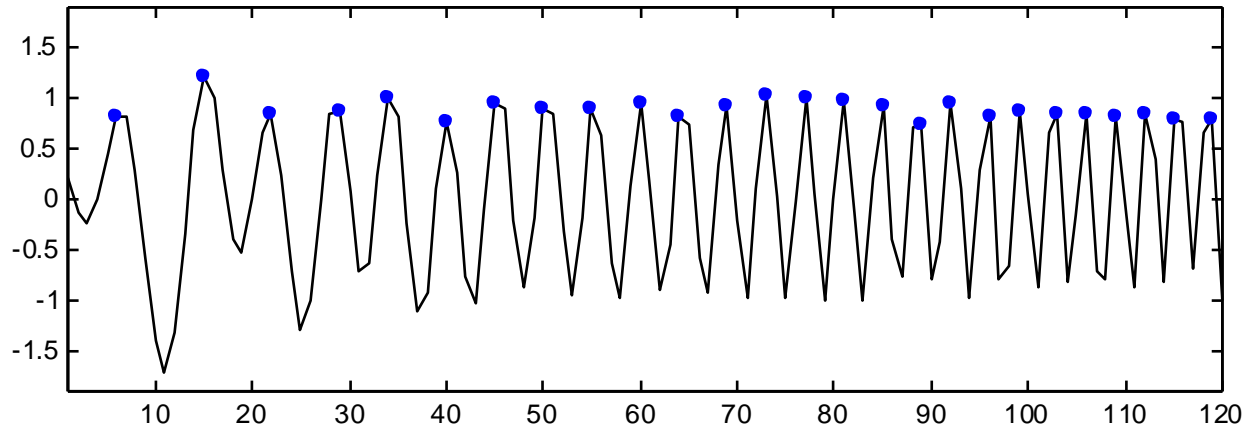
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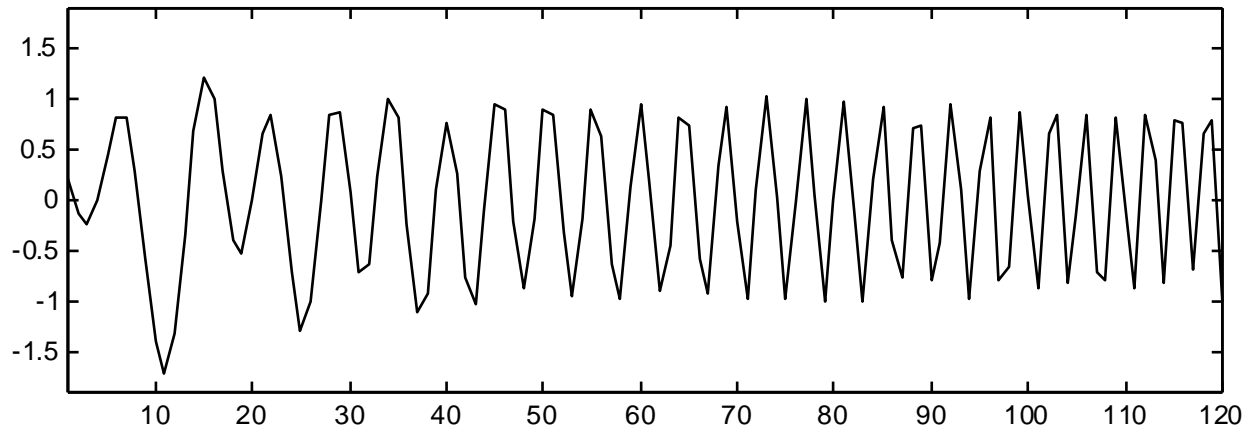
residue



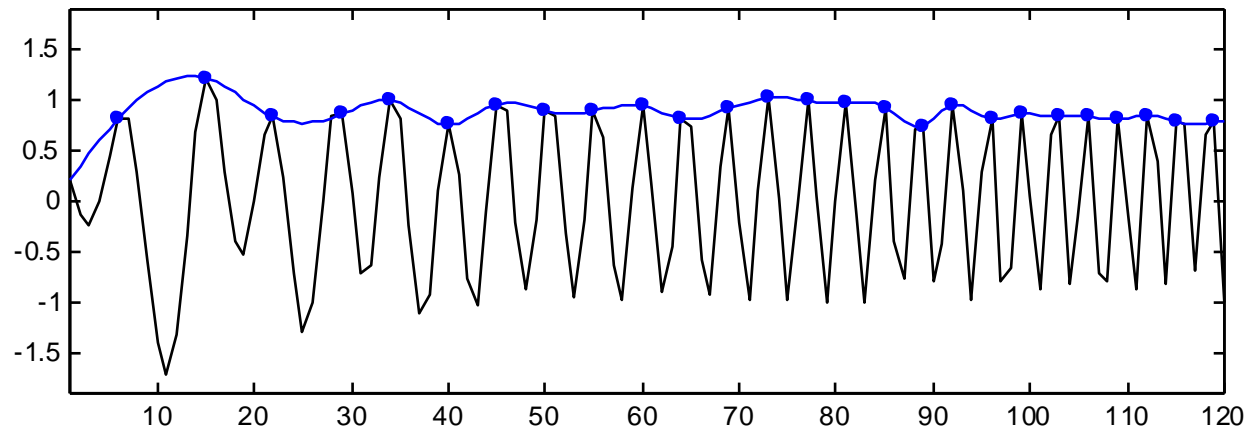
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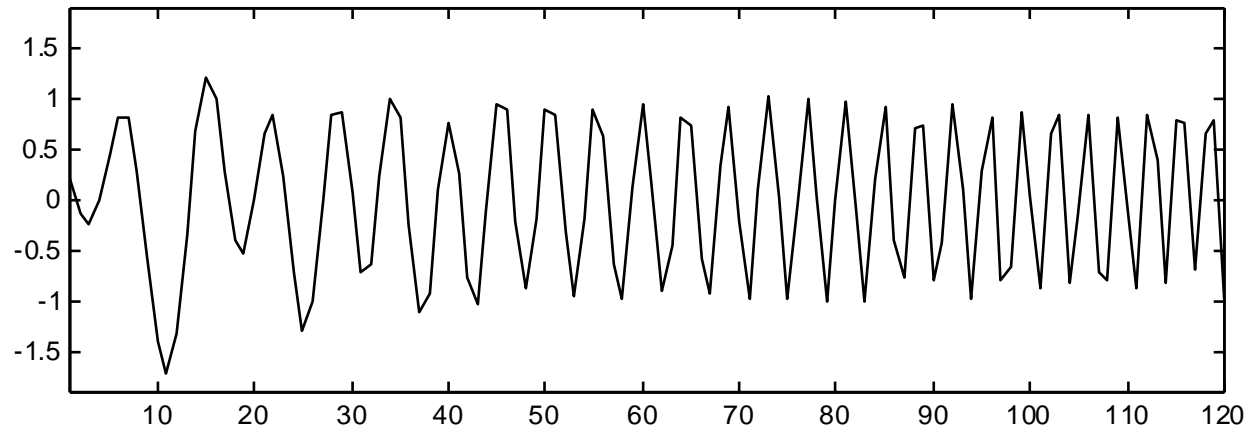
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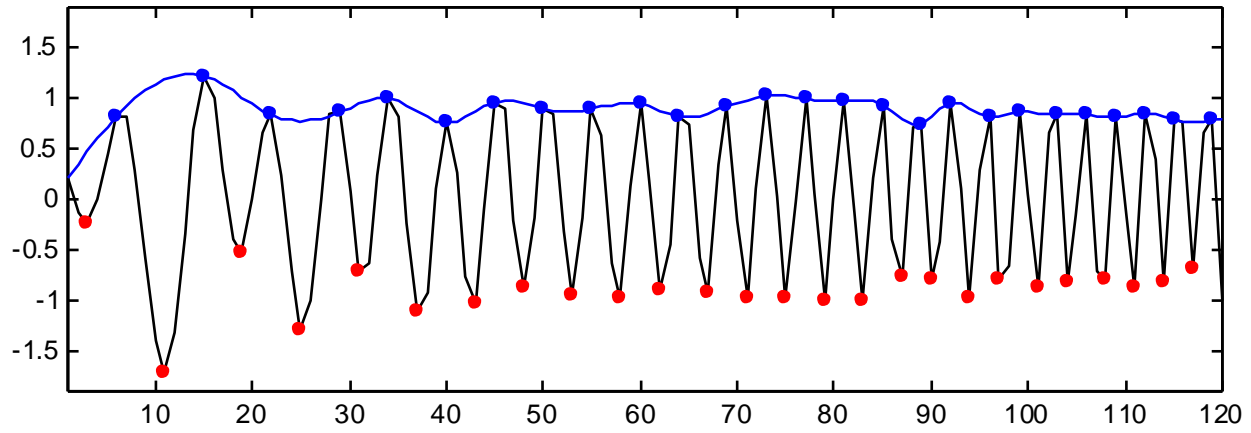
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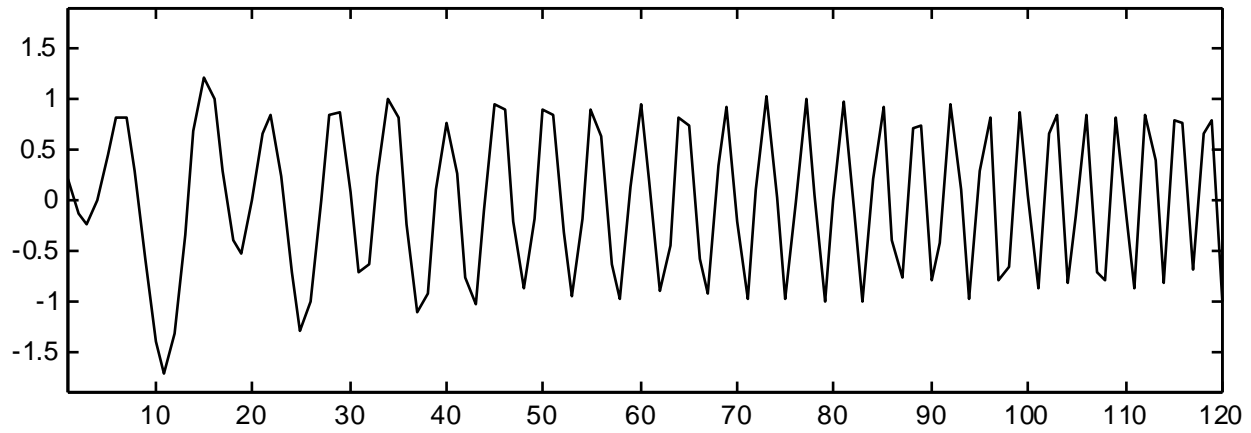
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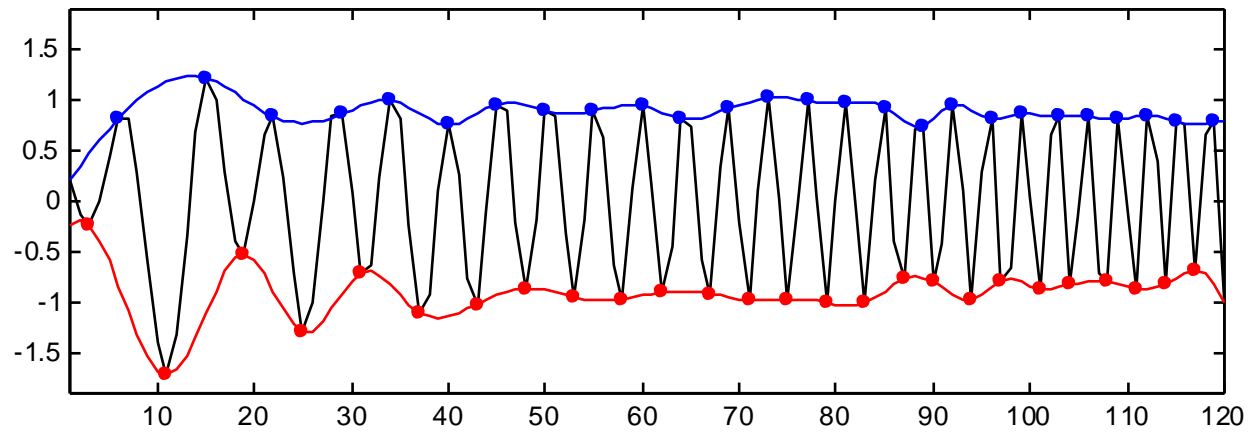
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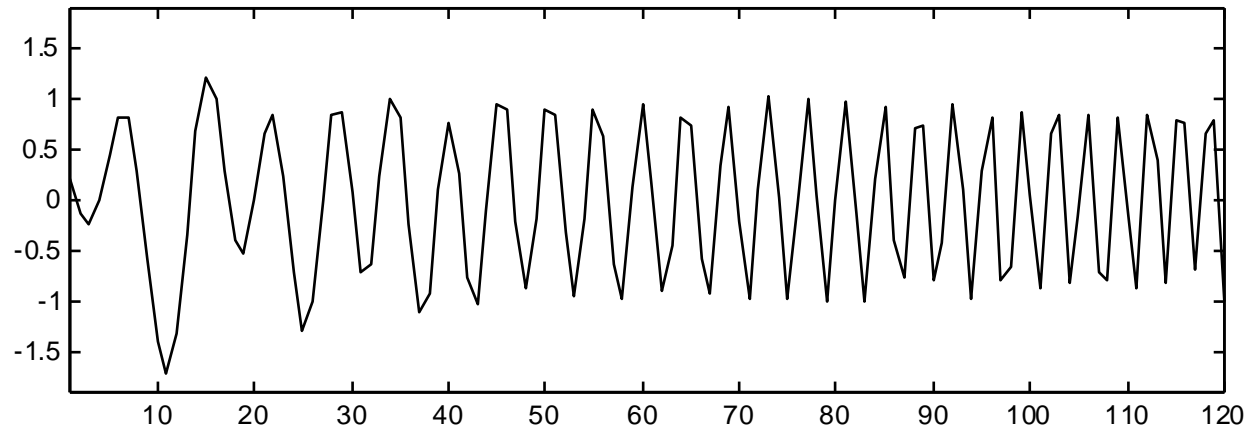
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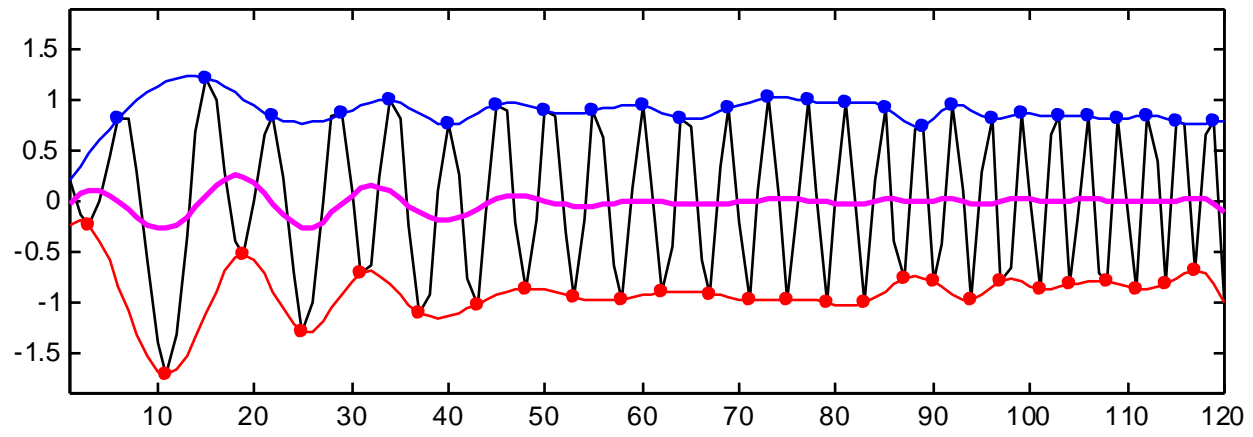
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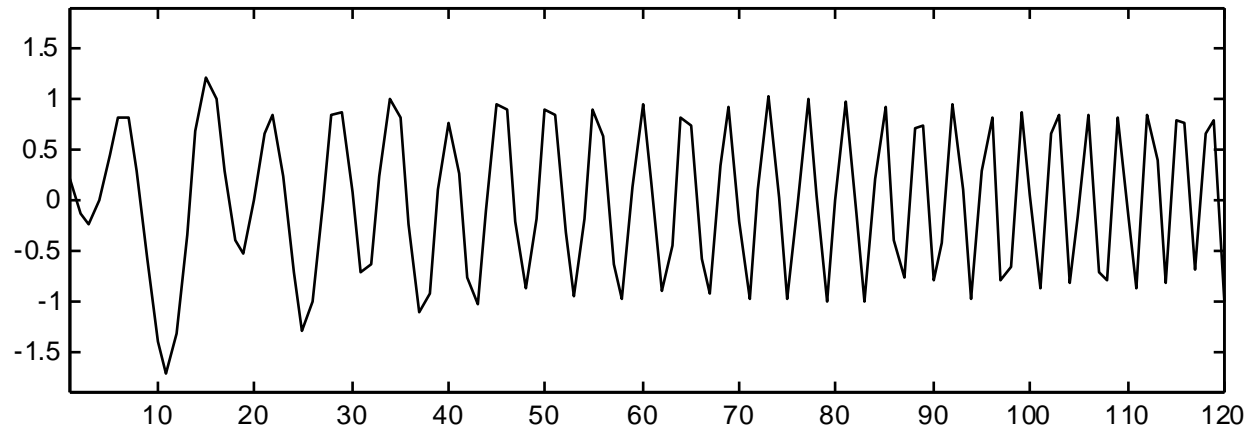
residue



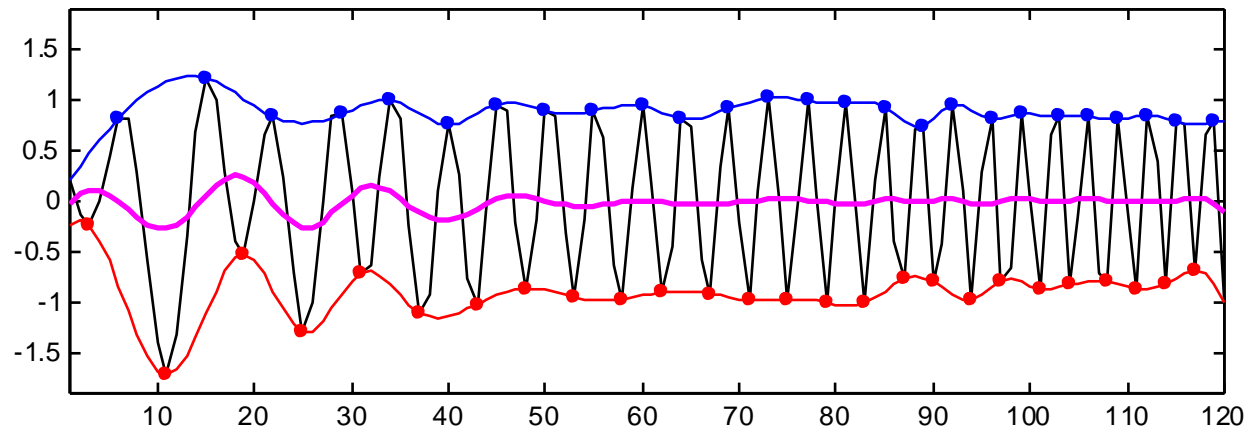
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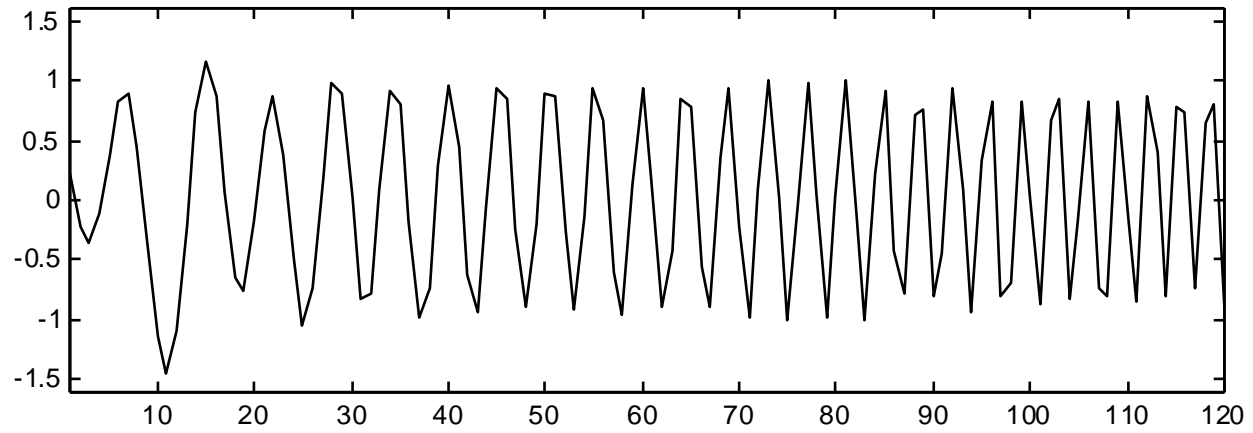
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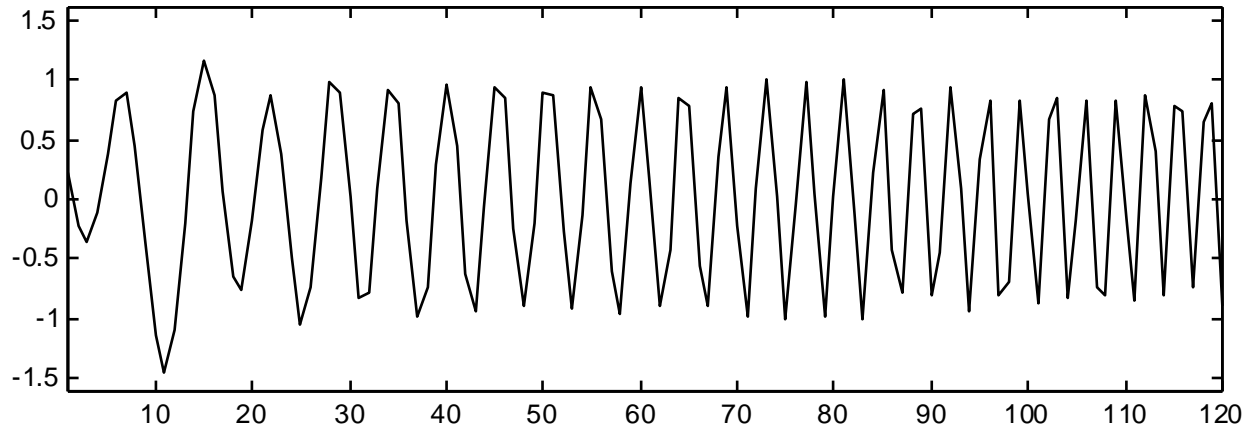
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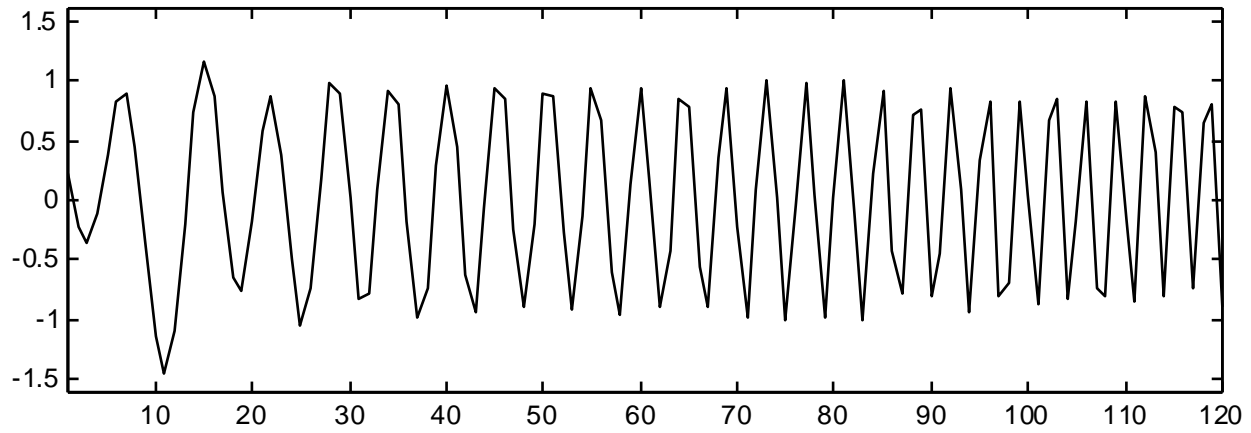
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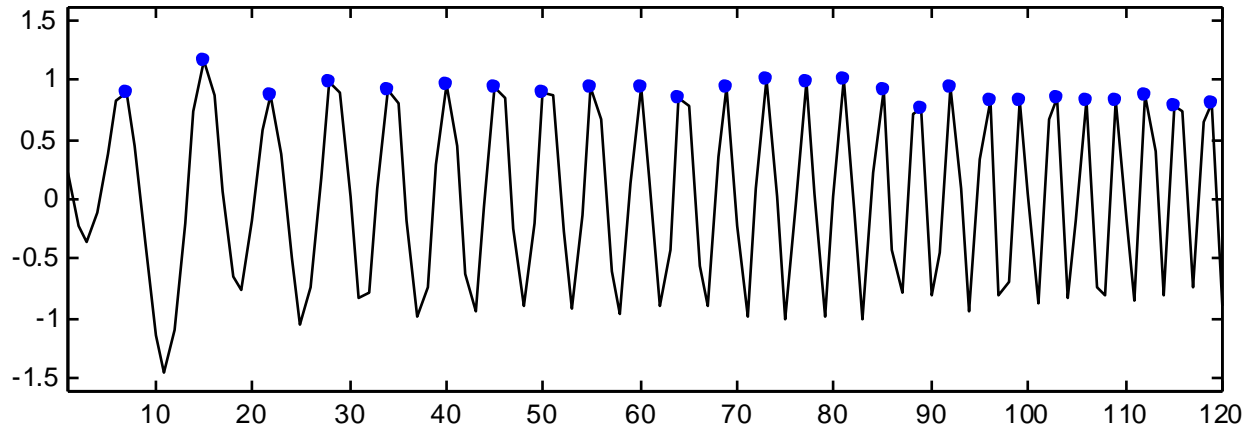
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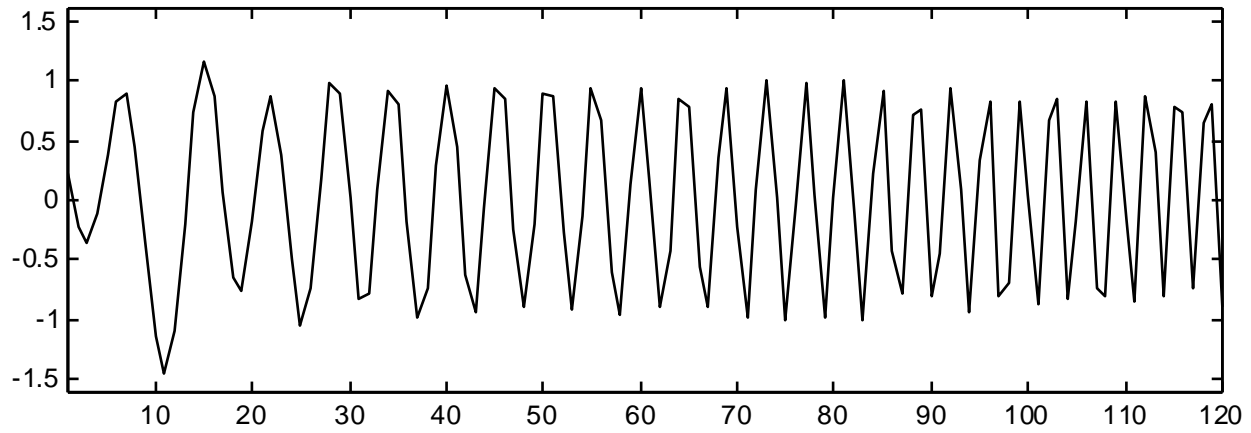
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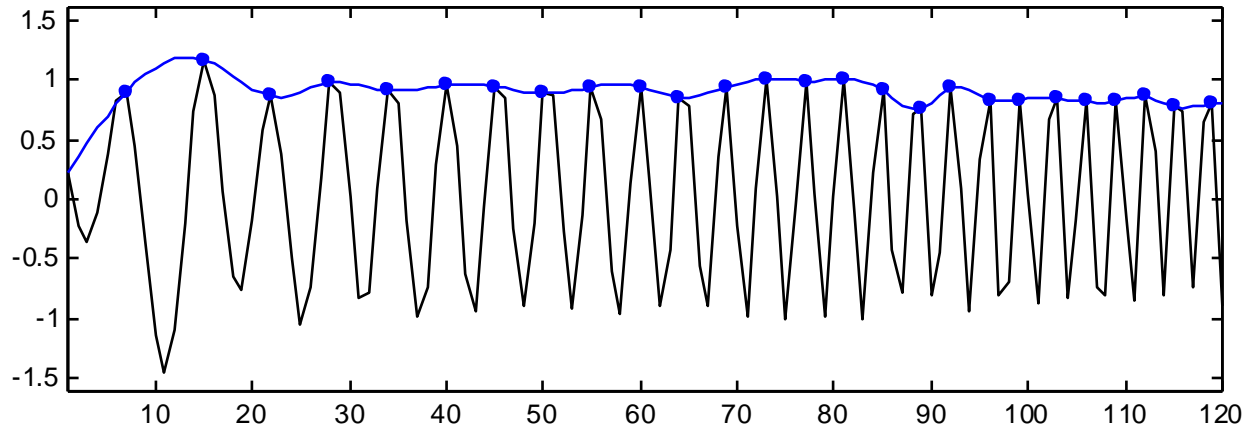
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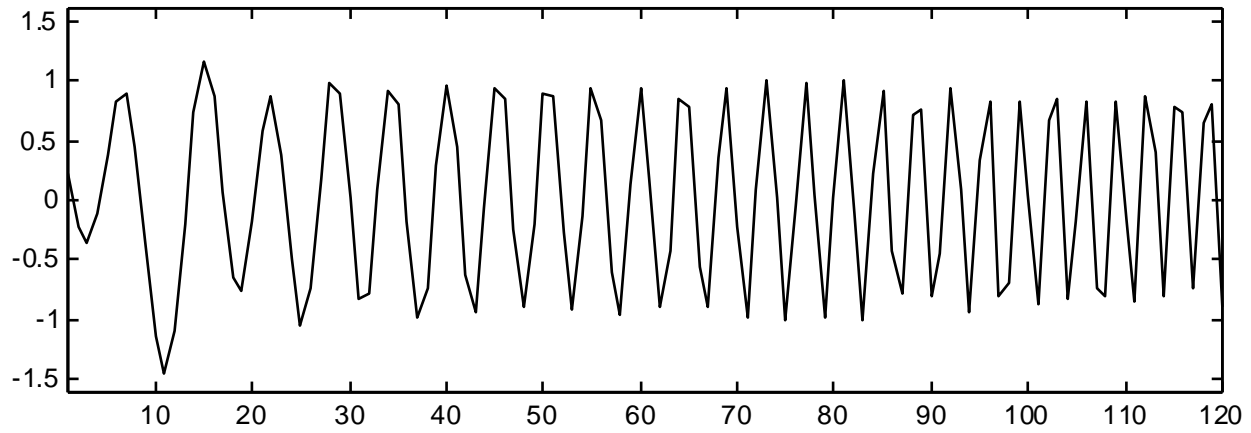
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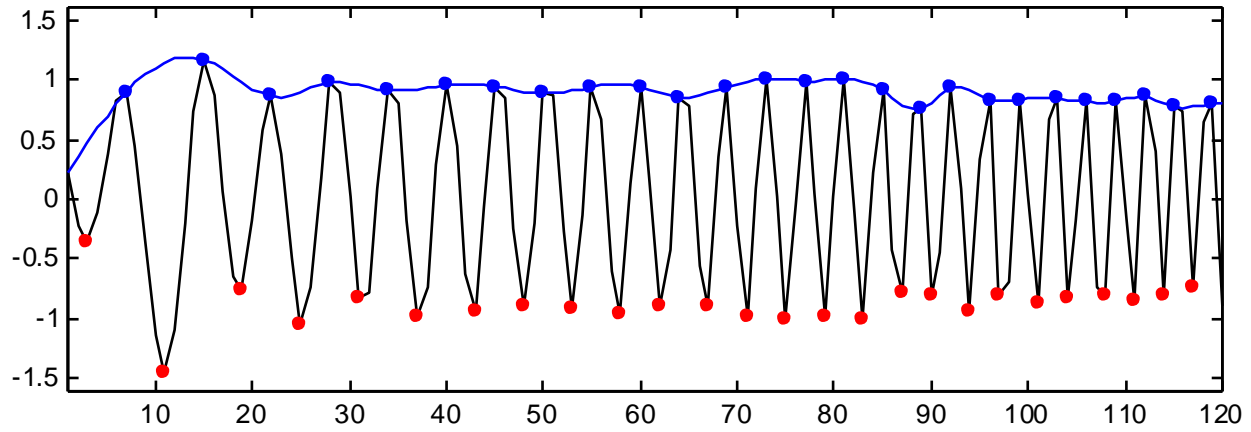
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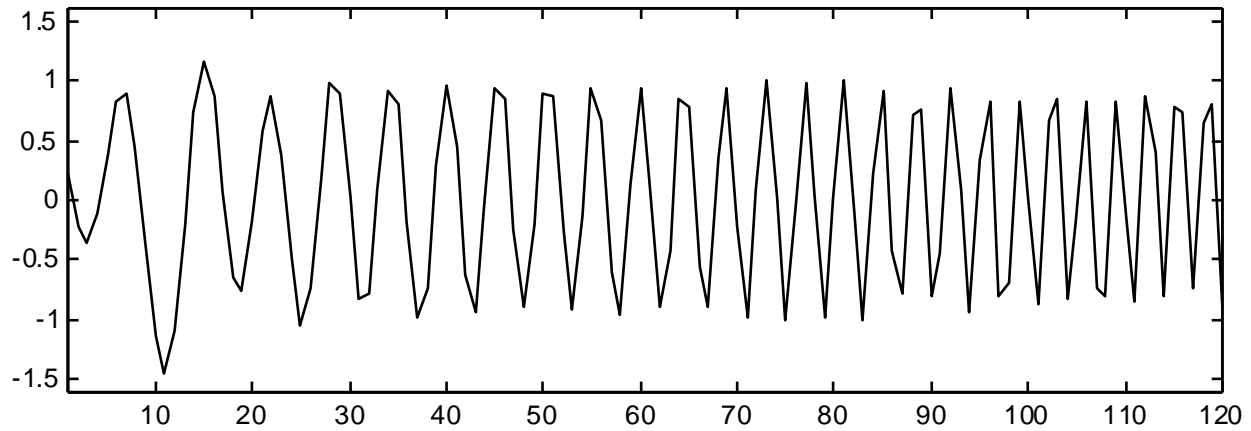
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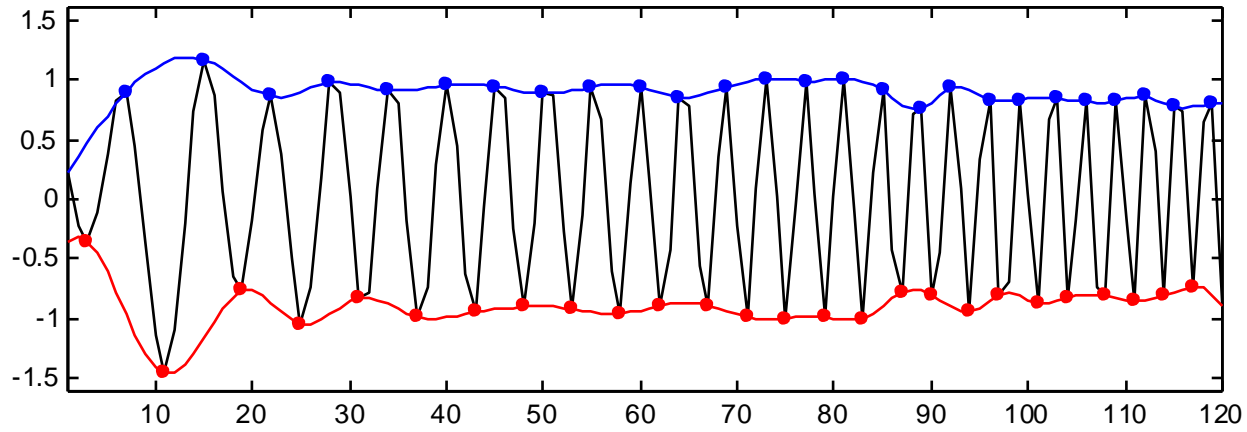
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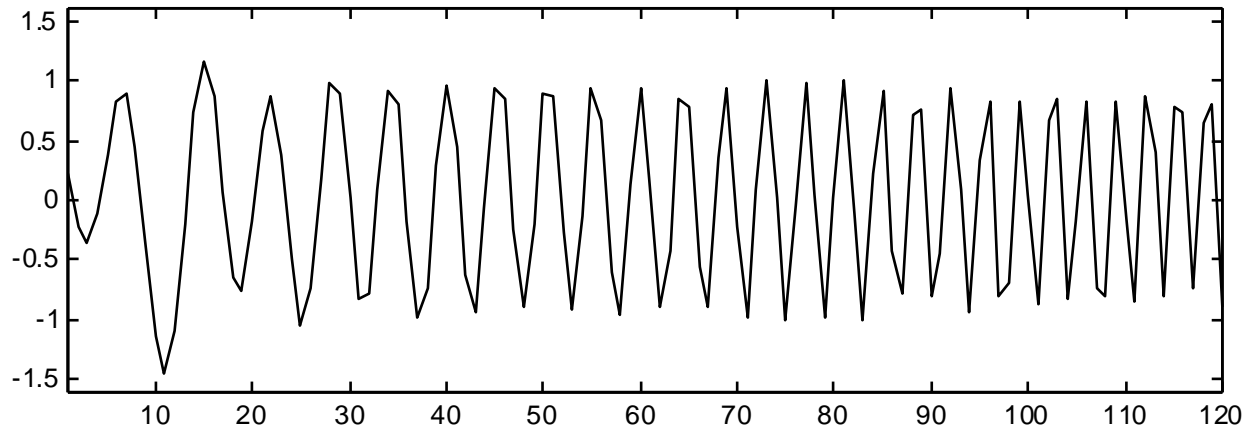
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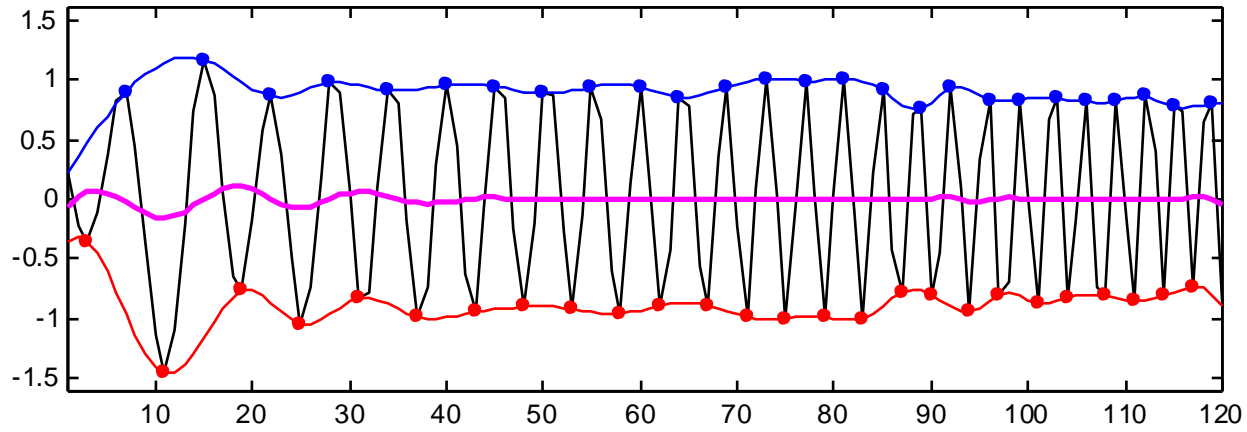
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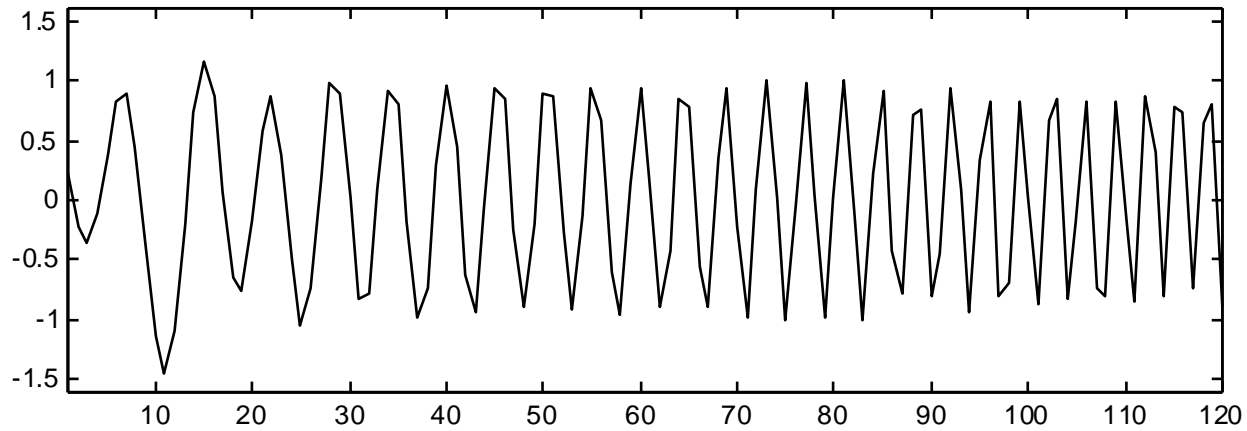
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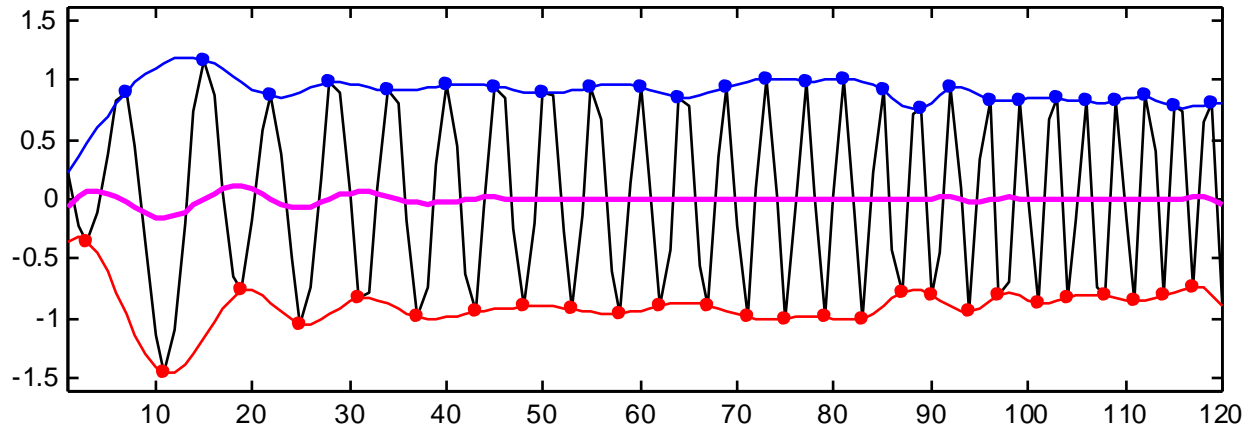
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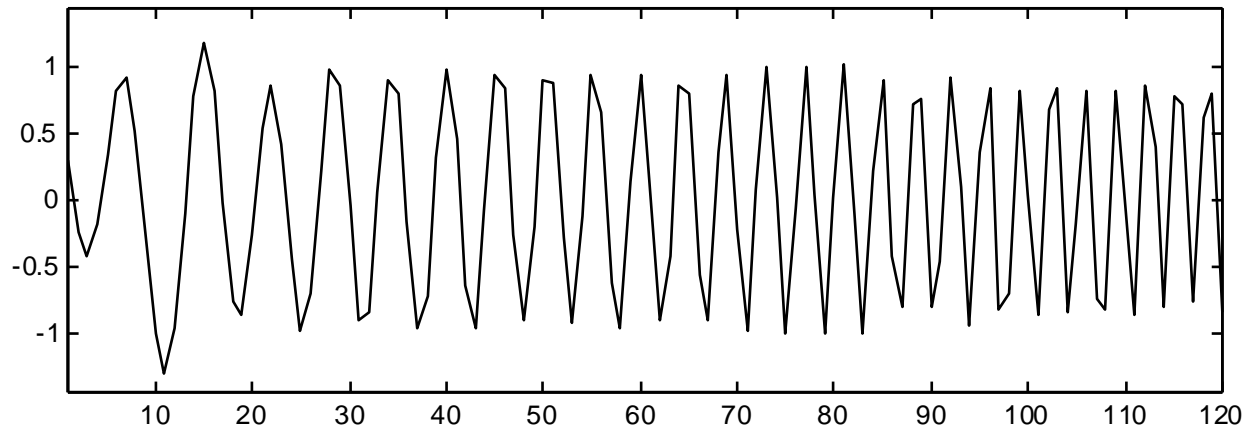
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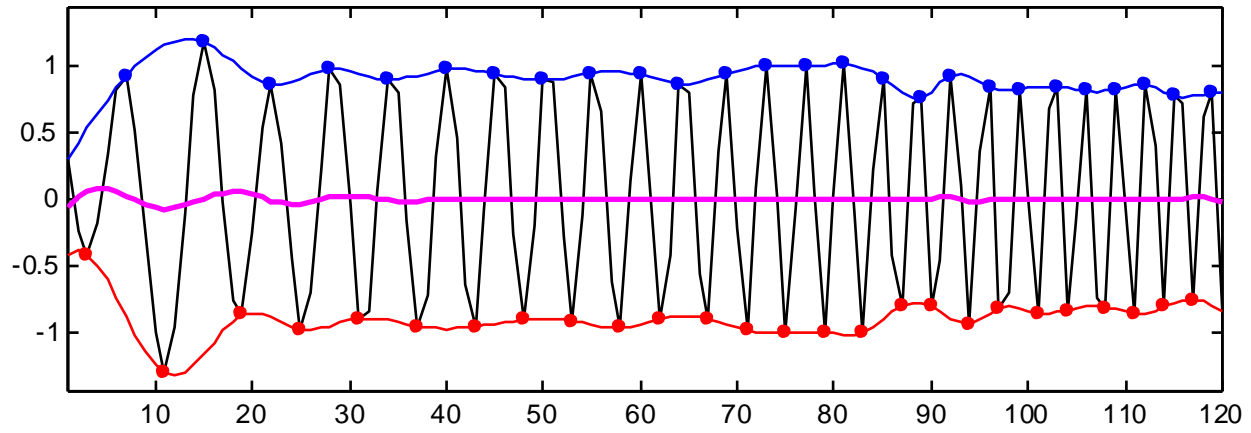
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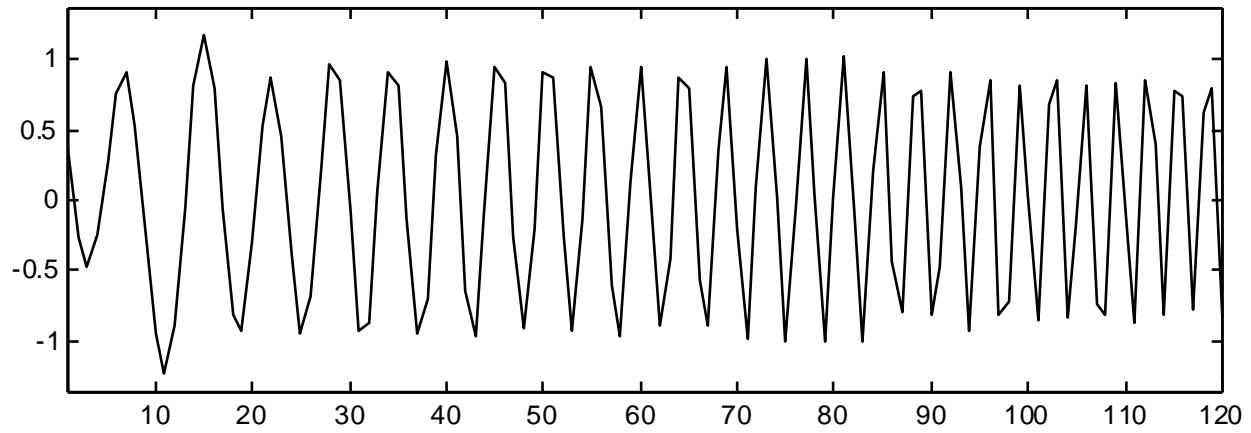
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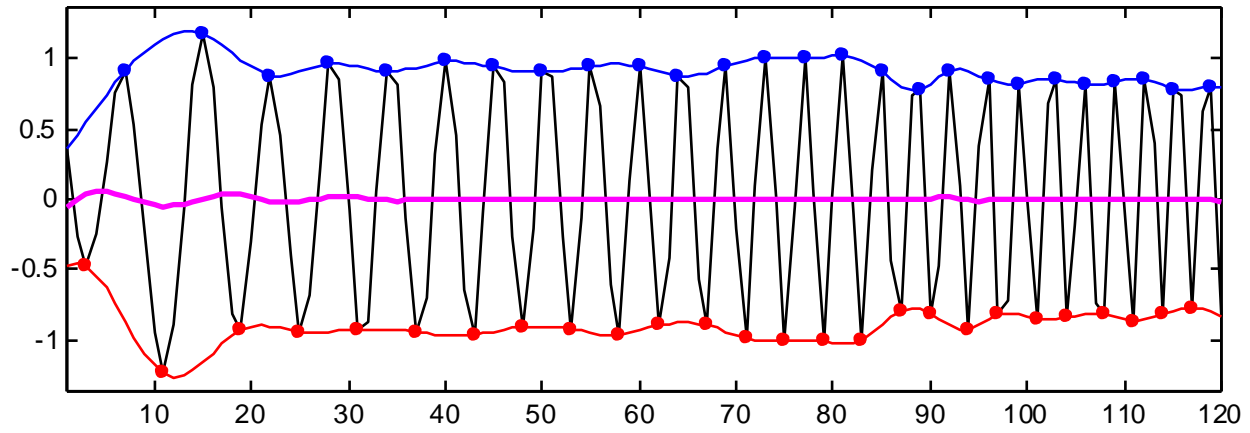
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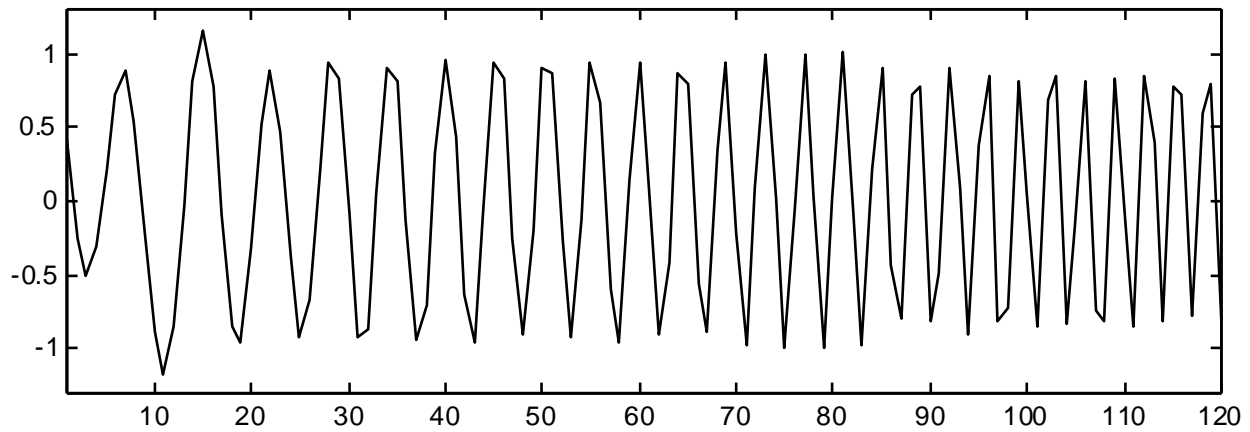
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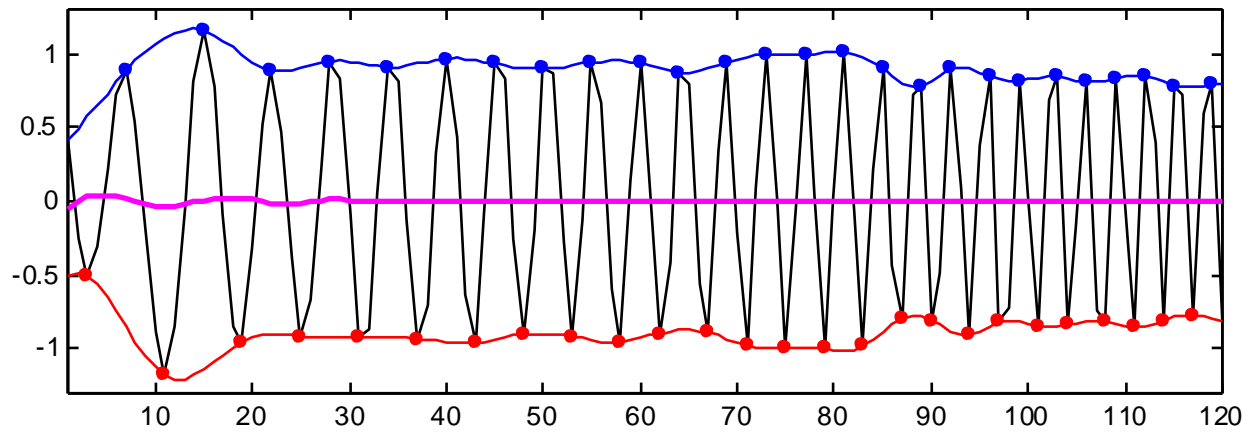
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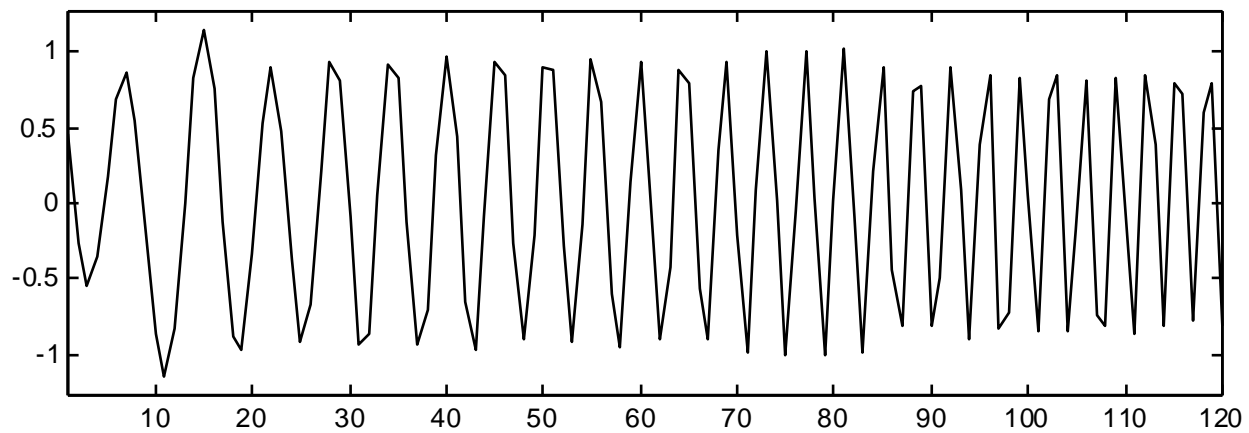
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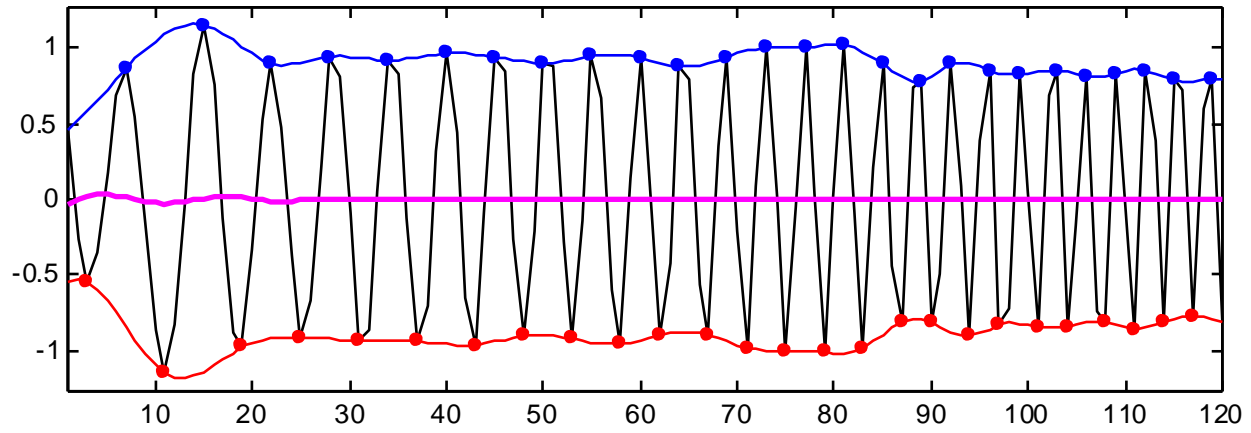
IMF 1; iteration 5



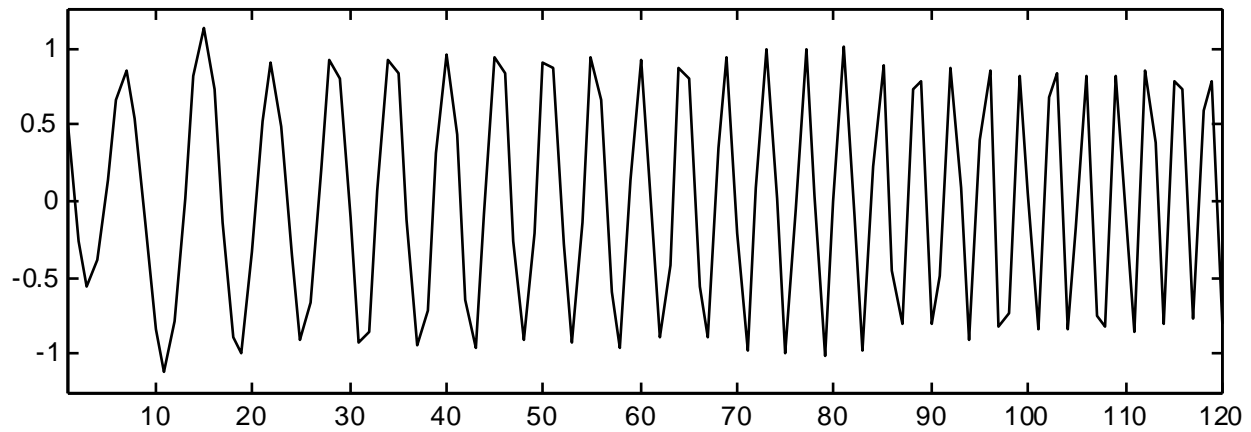
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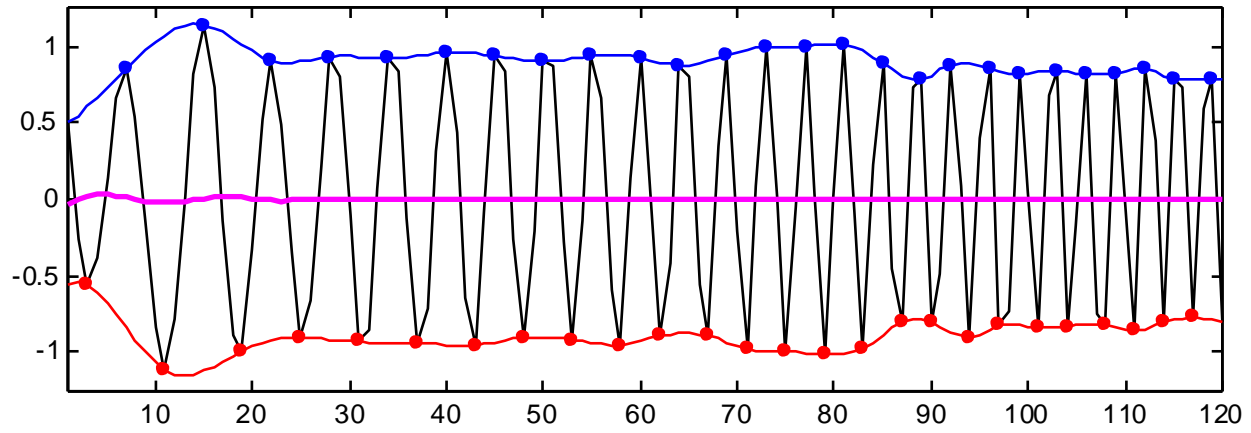
IMF 1; iteration 6



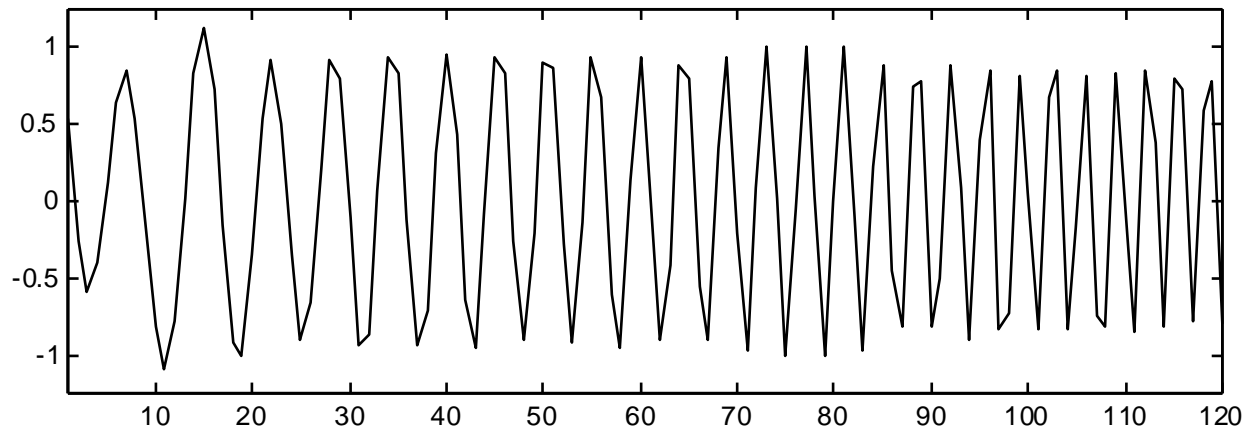
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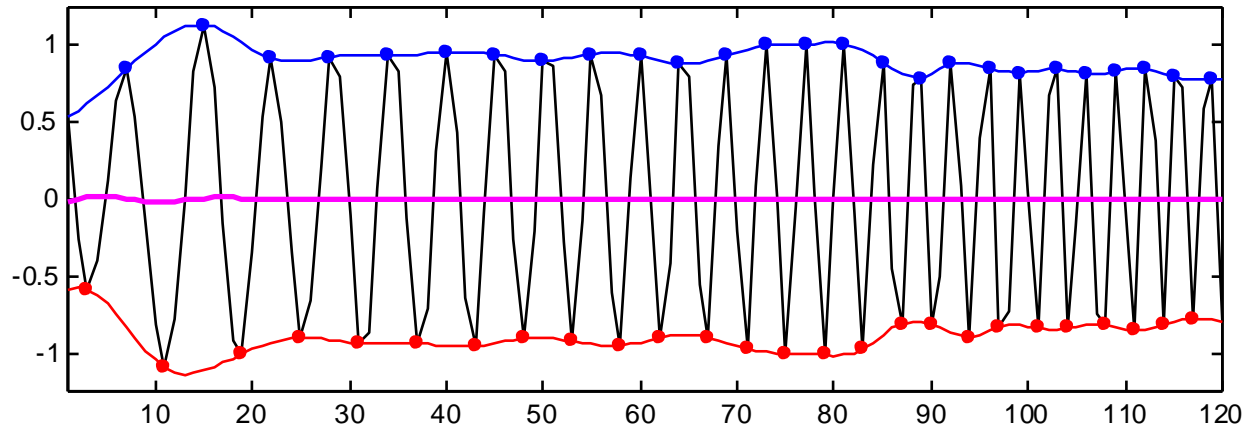
IMF 1; iteration 7



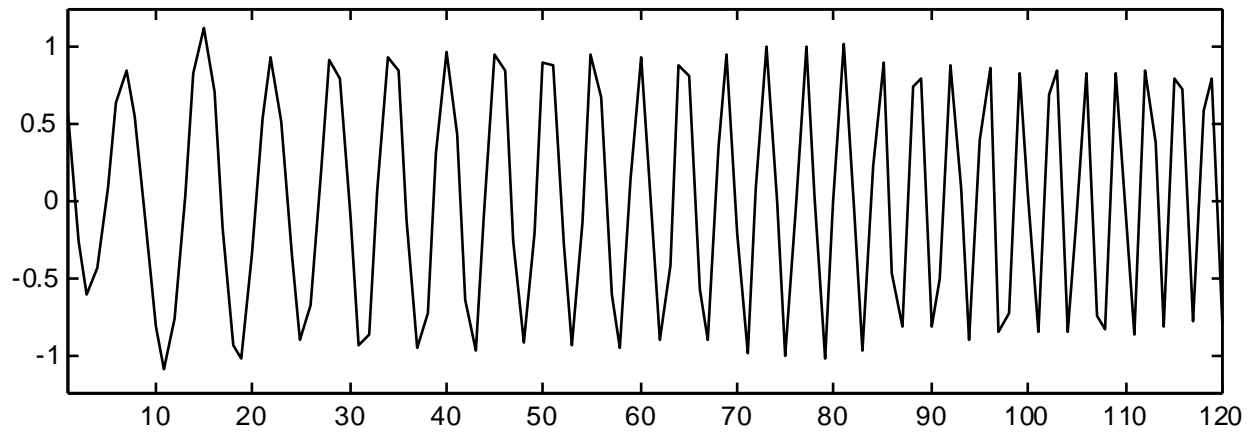
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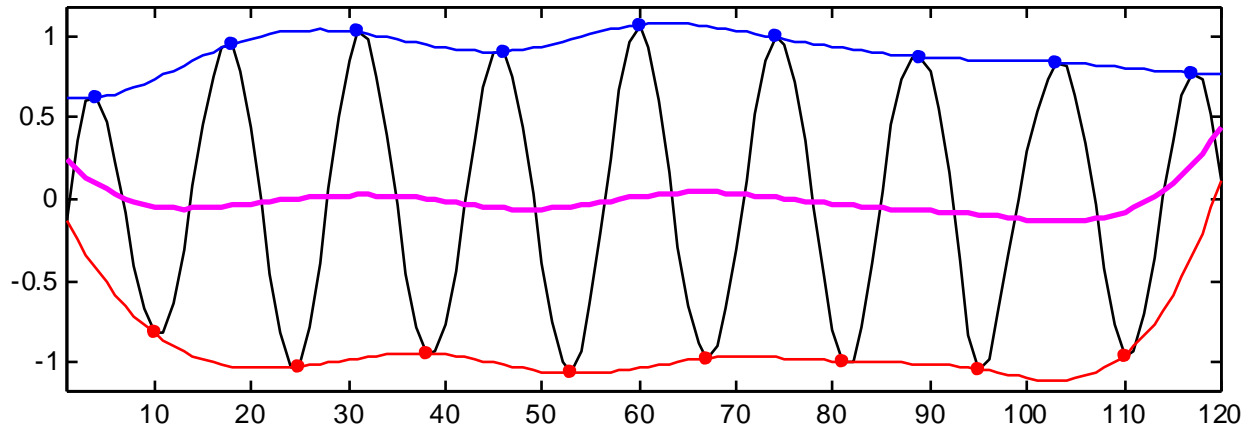
IMF 1; iteration 8



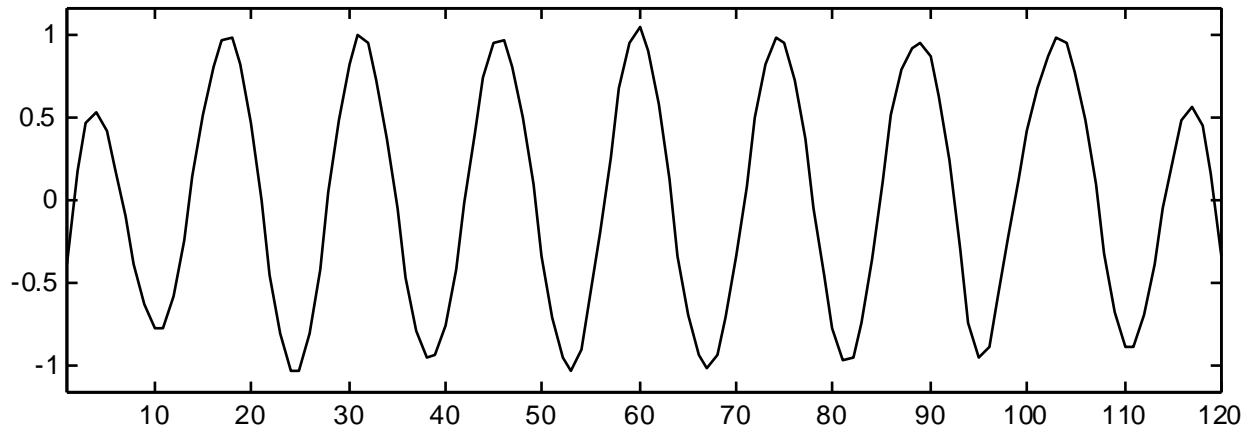
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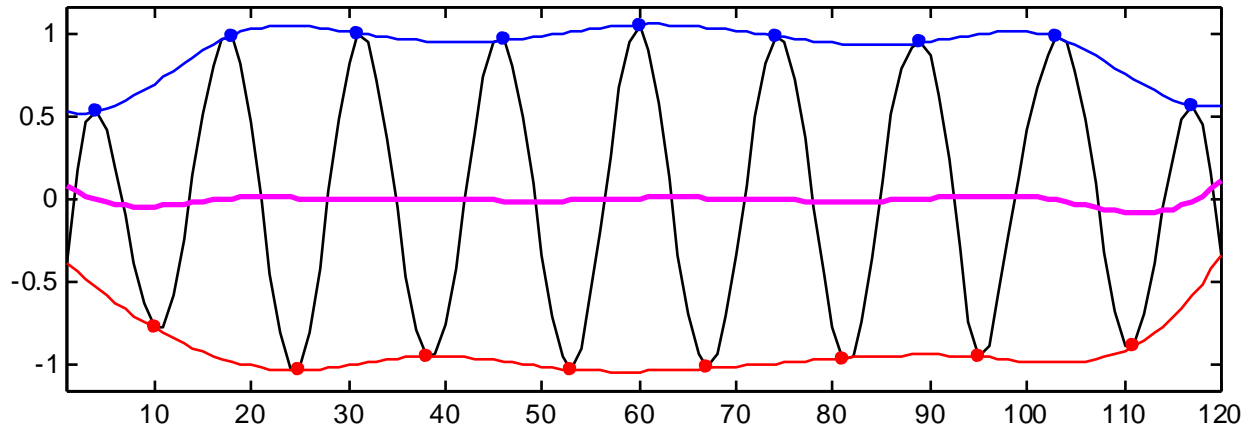
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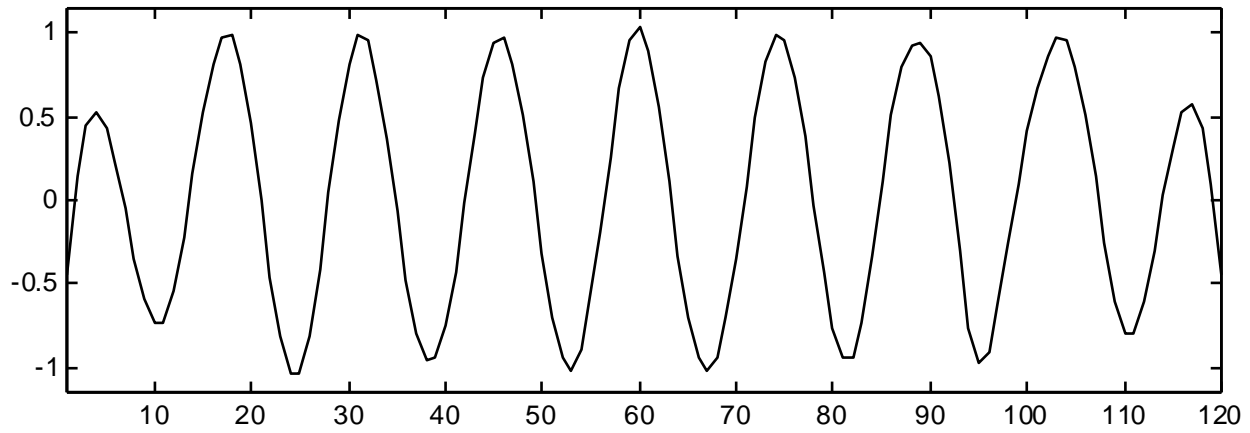
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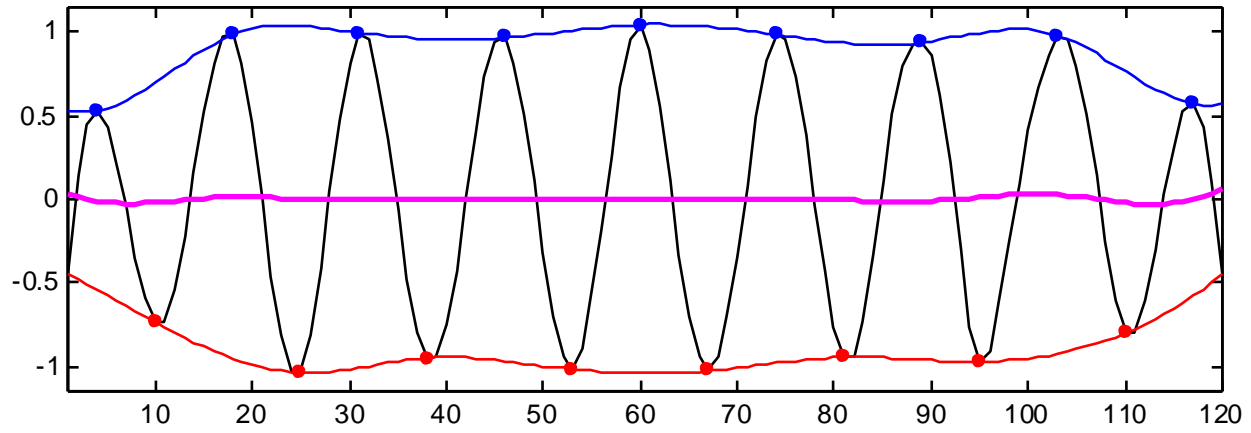
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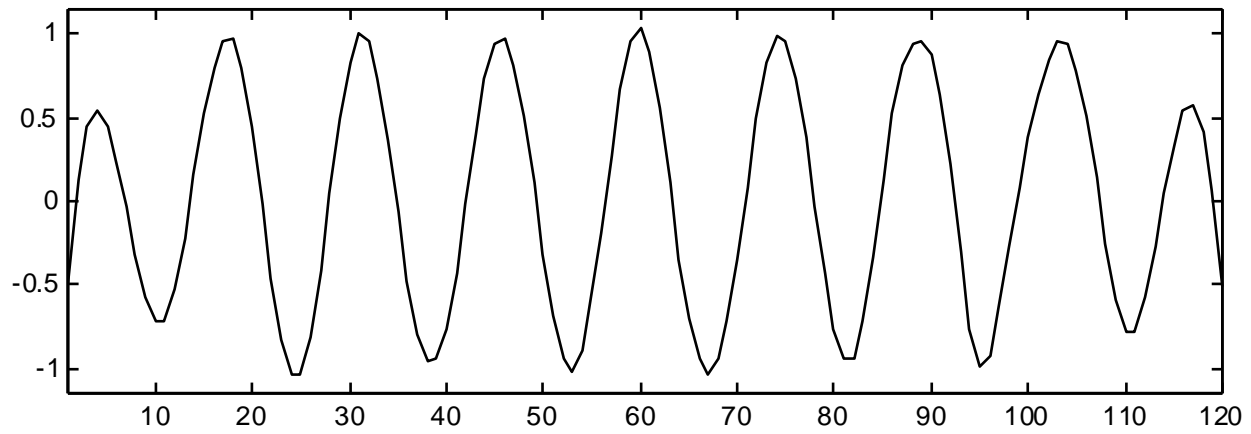
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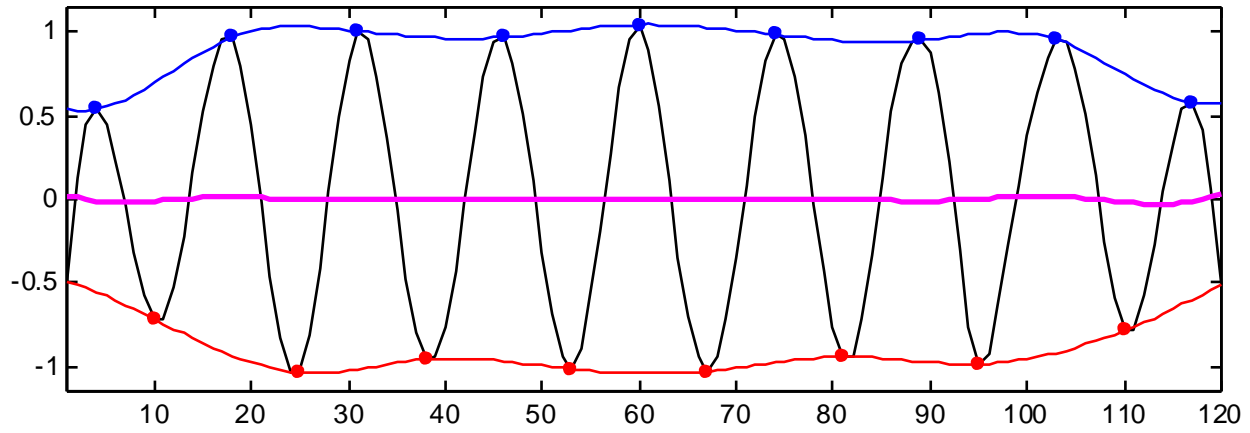
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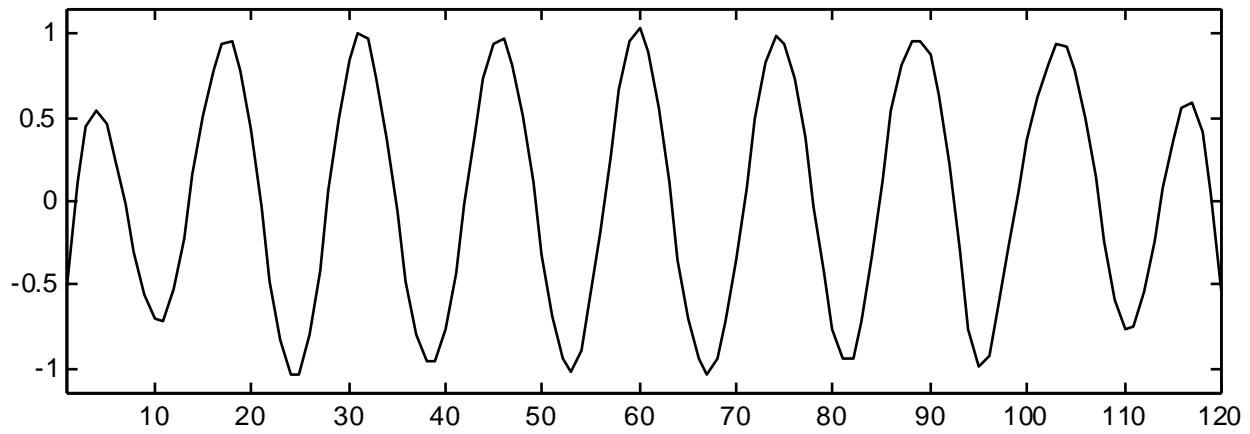
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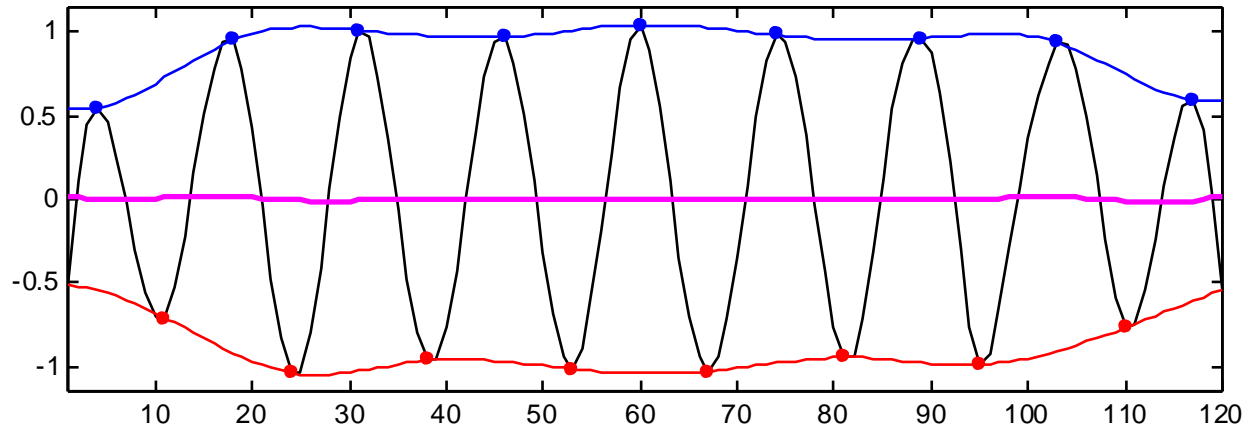
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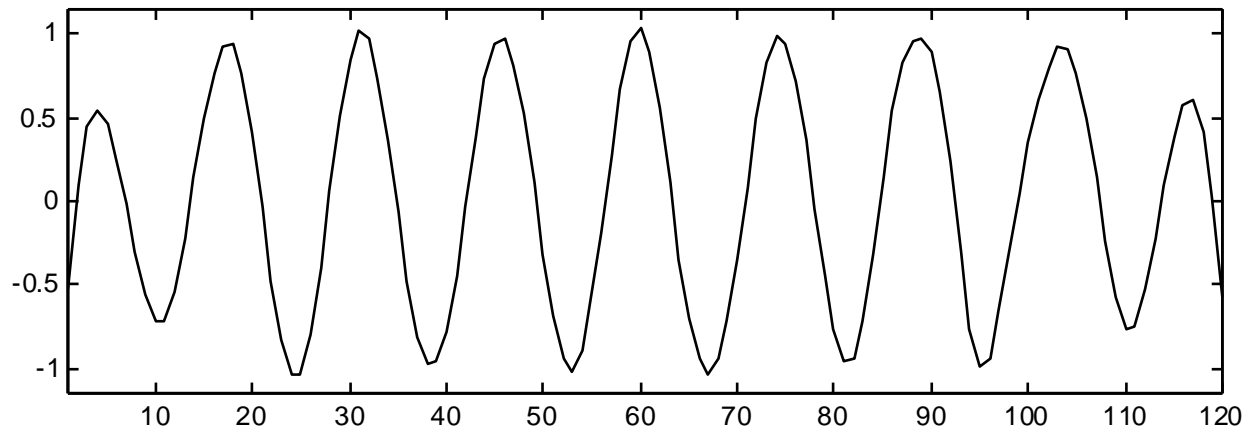
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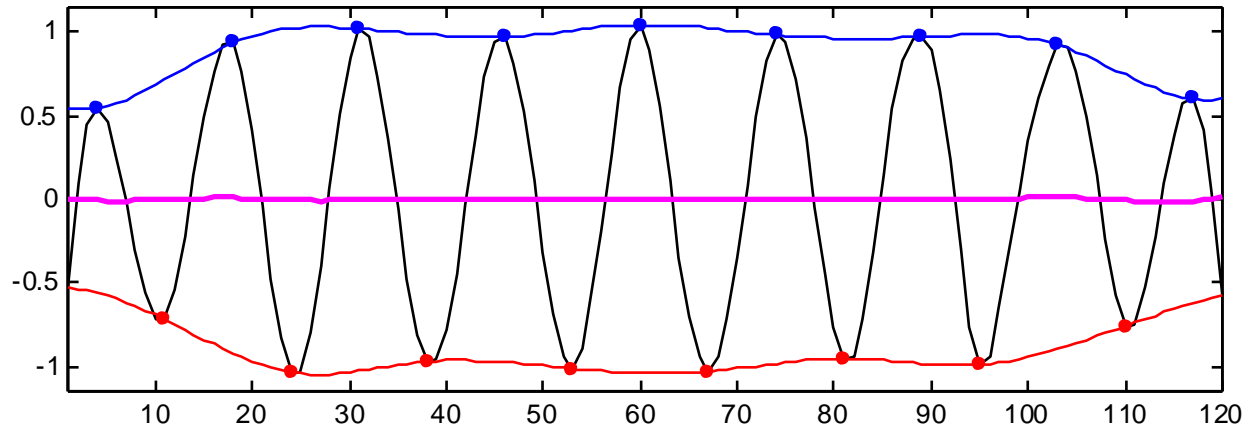
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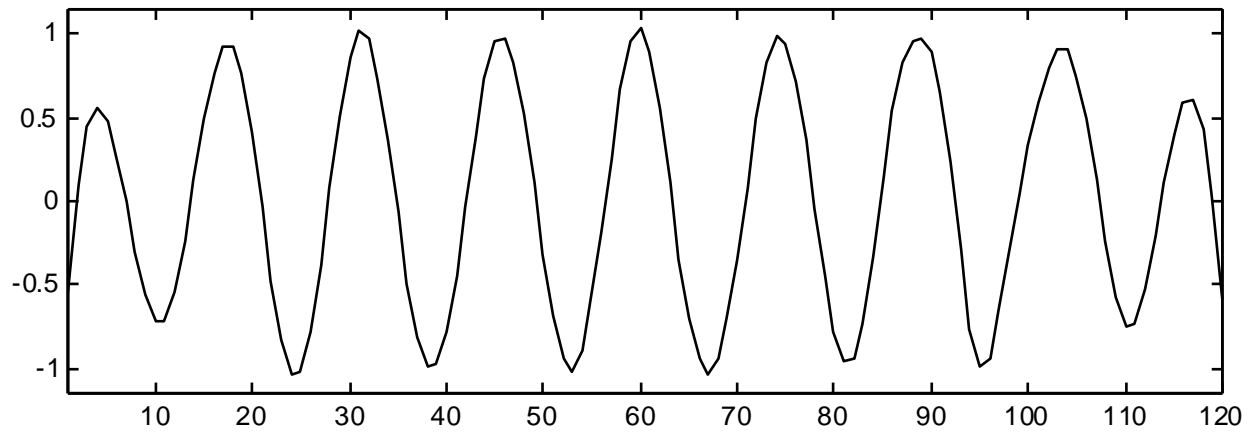
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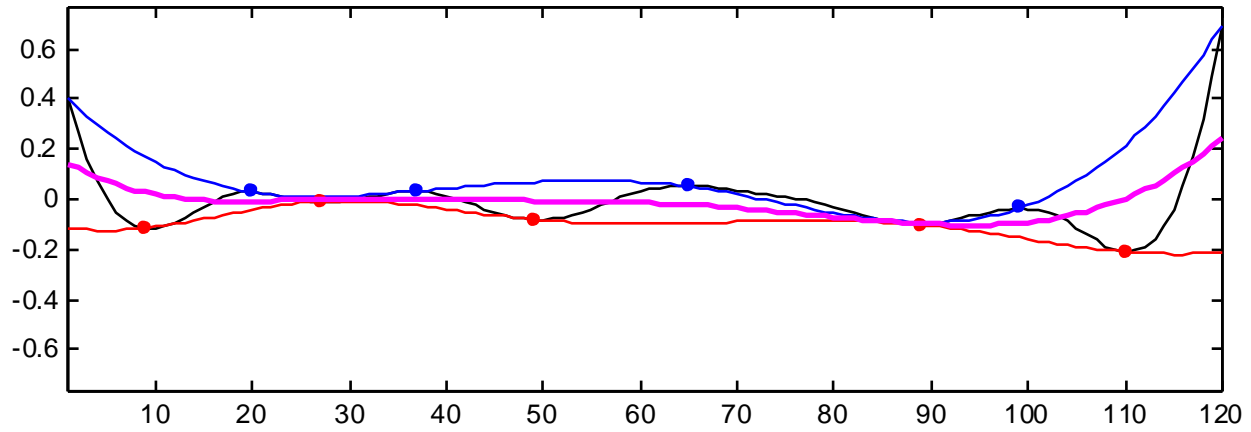
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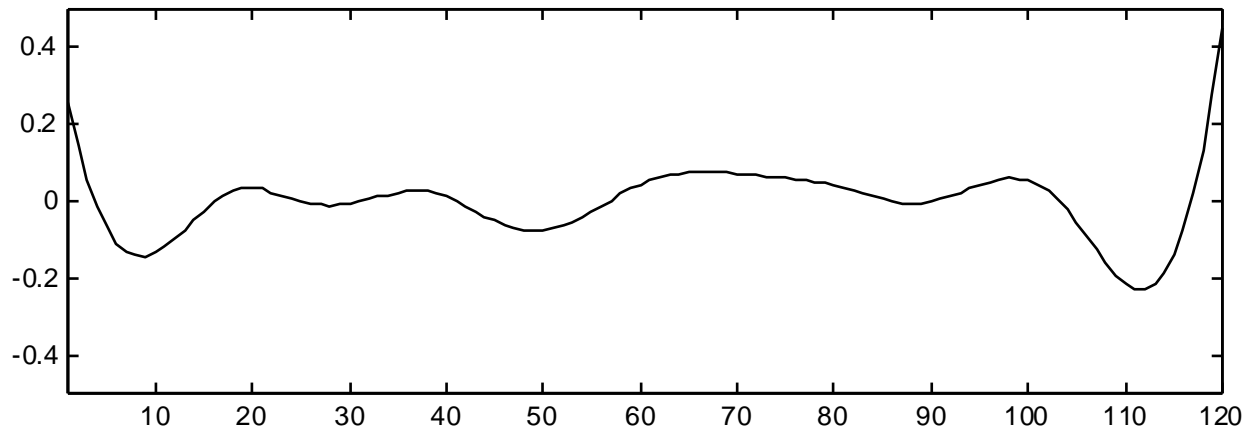
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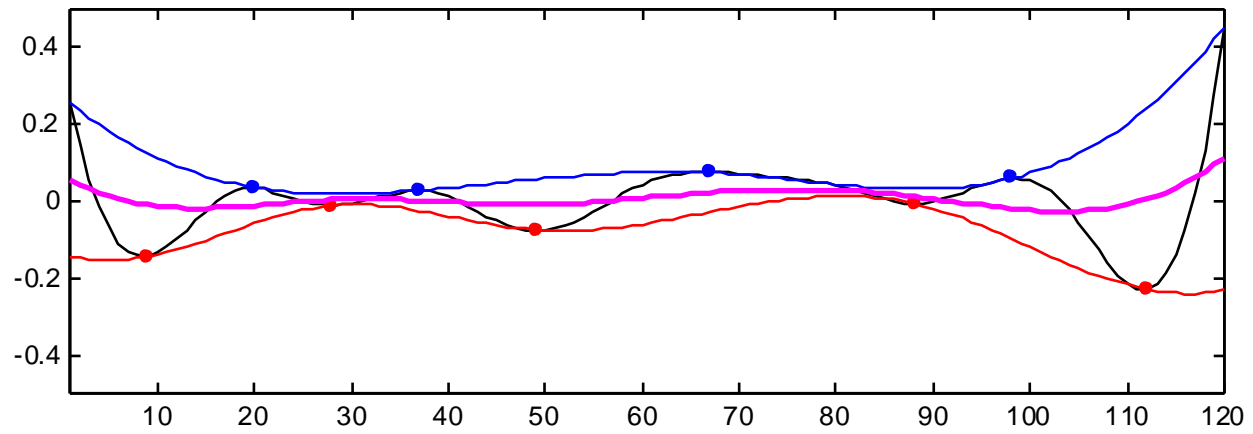
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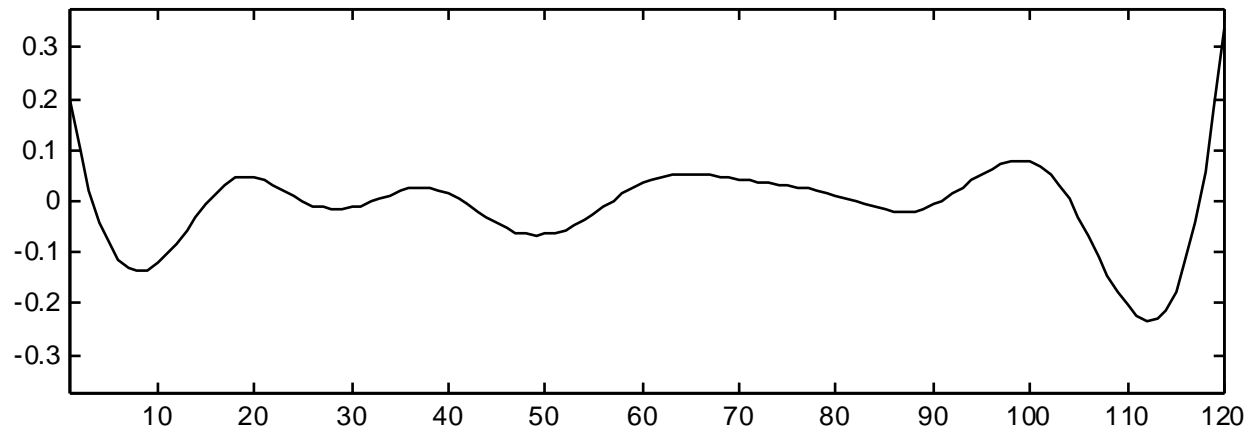
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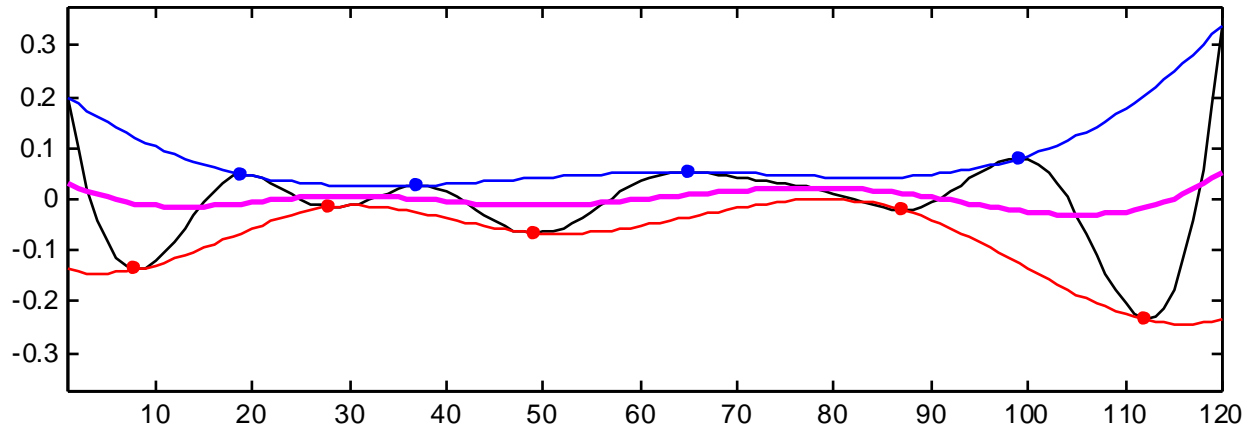
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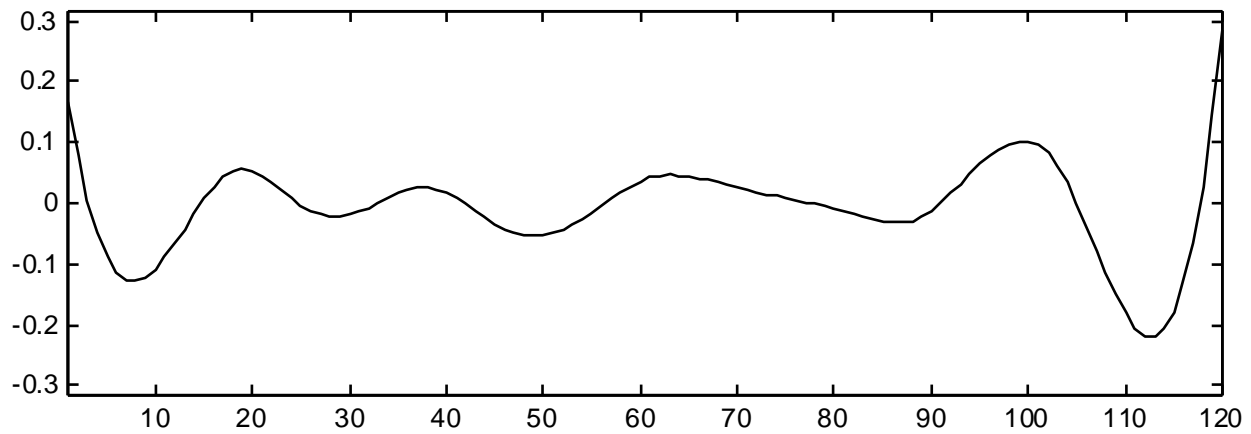
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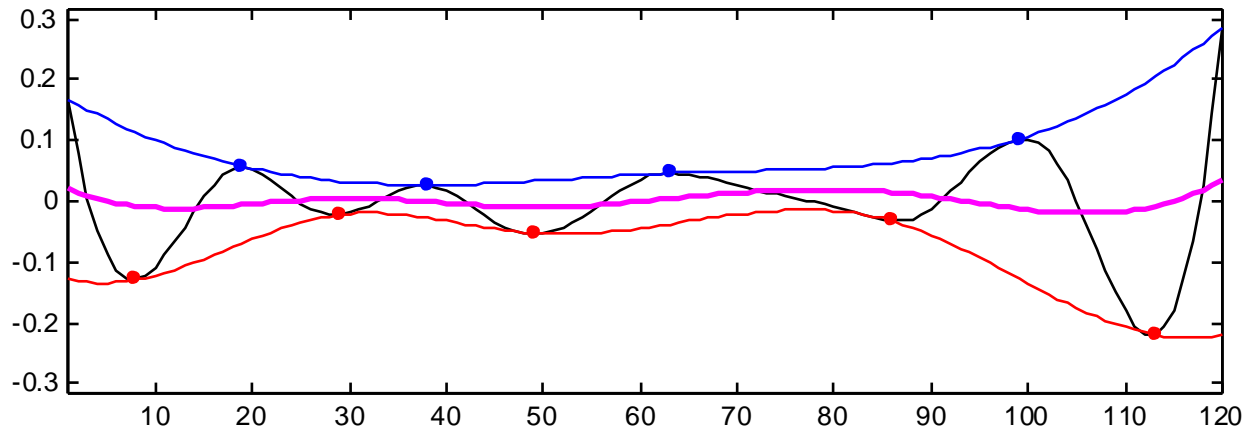
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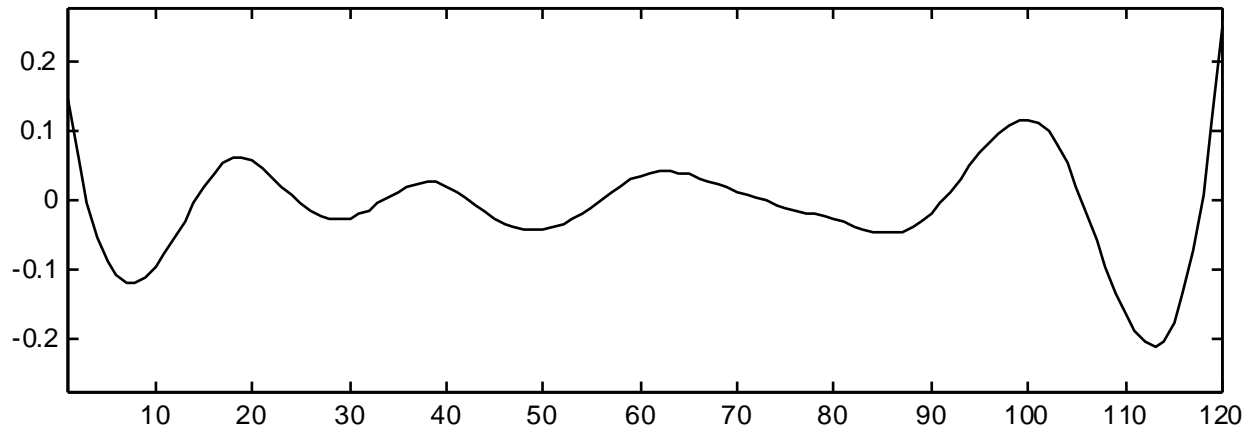
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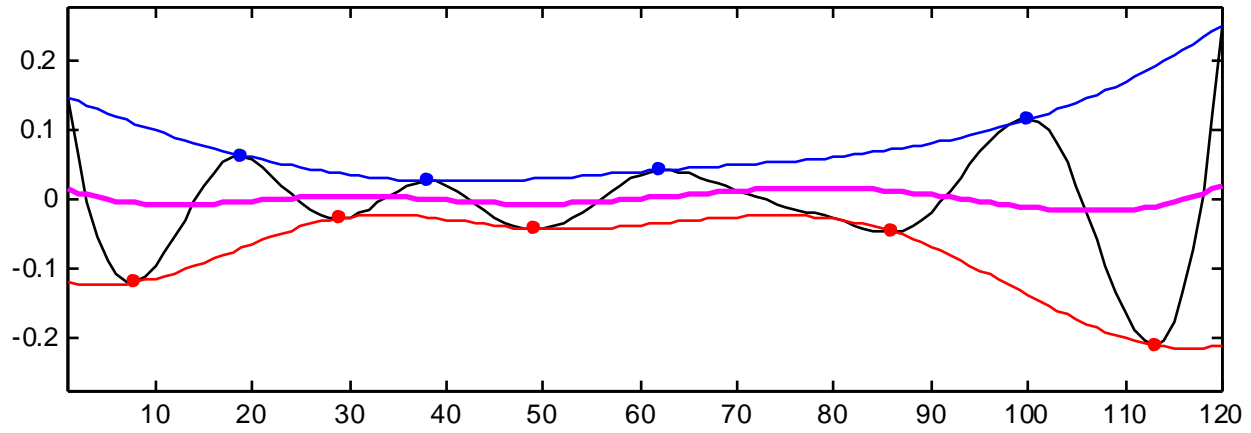
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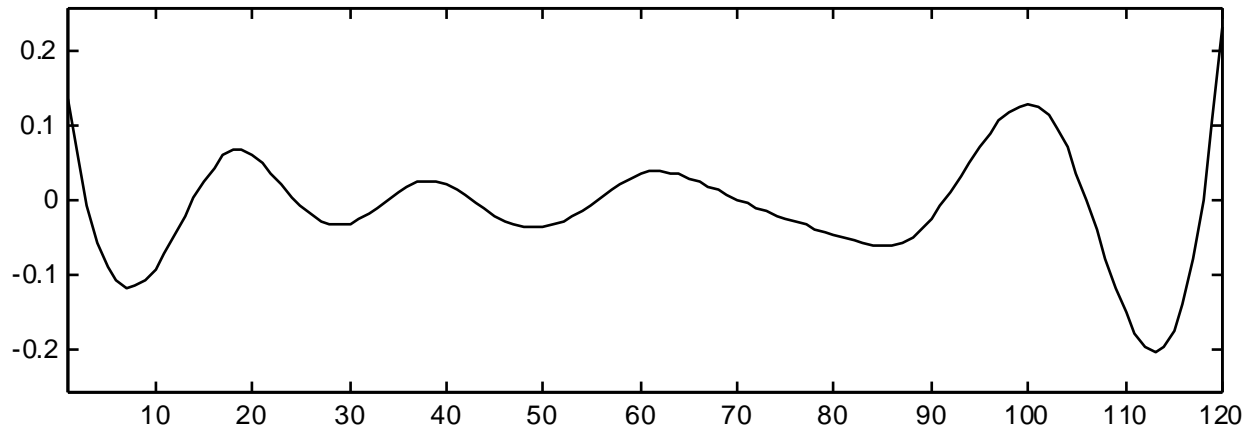
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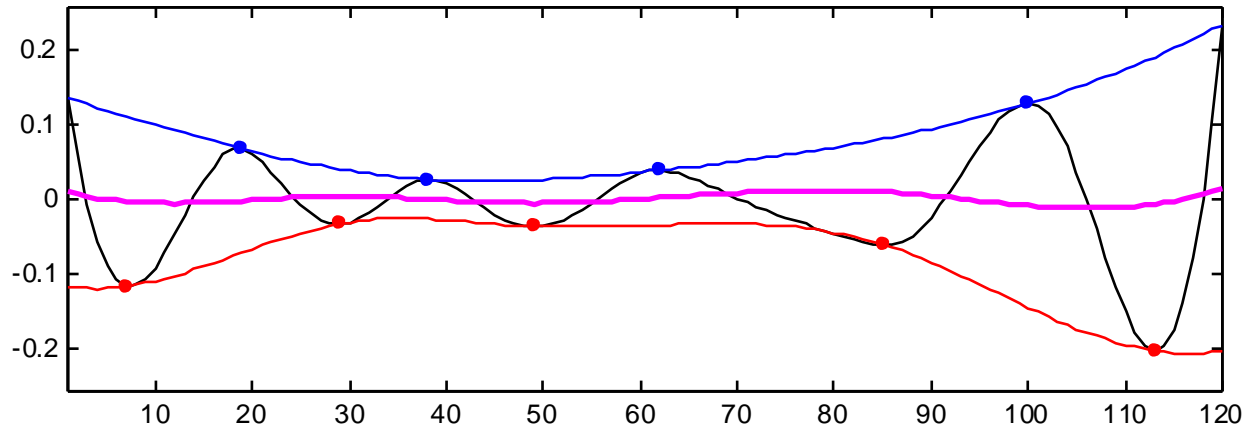
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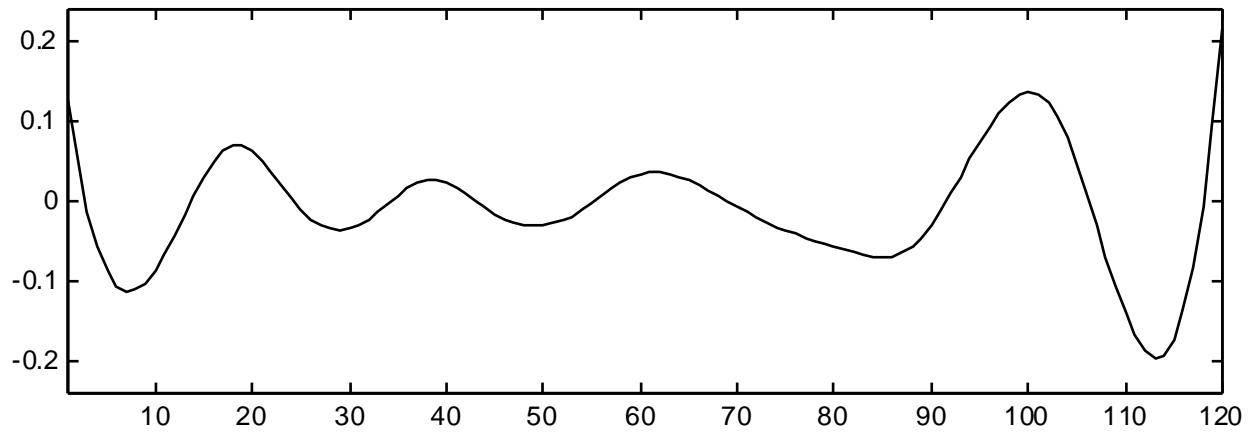
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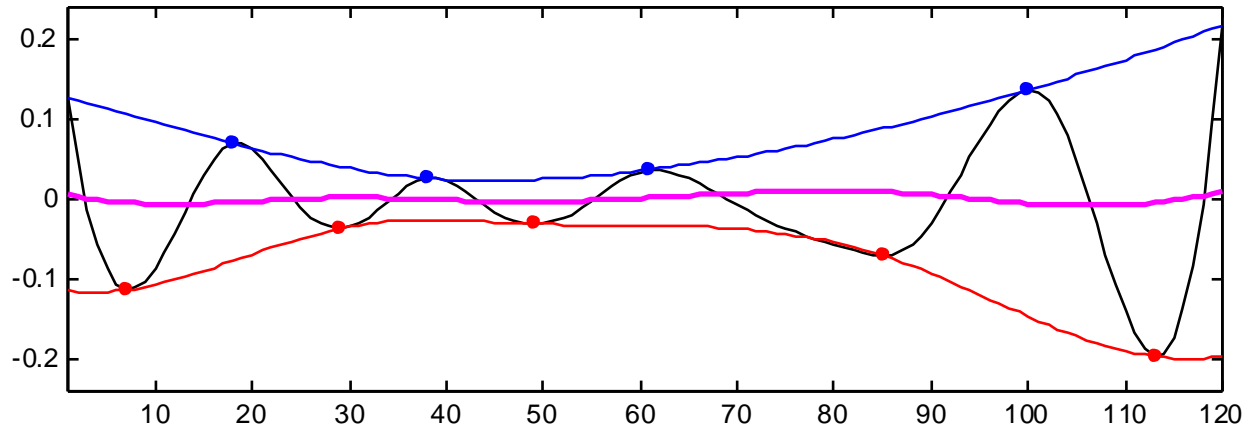
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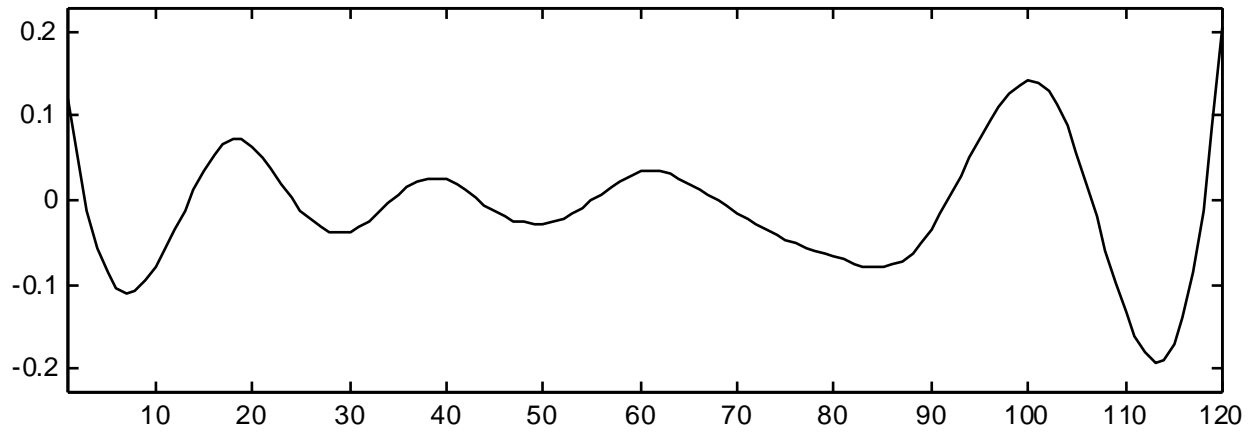
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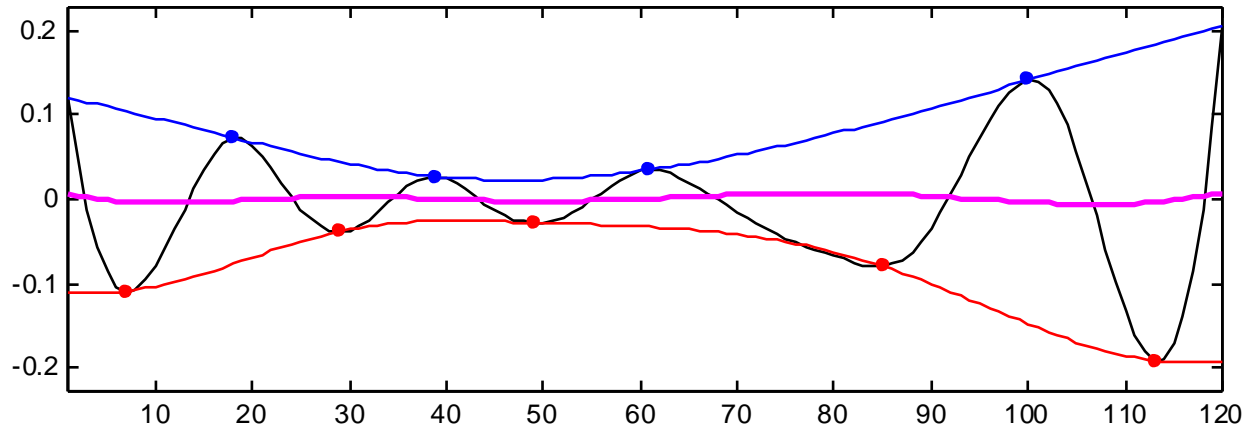
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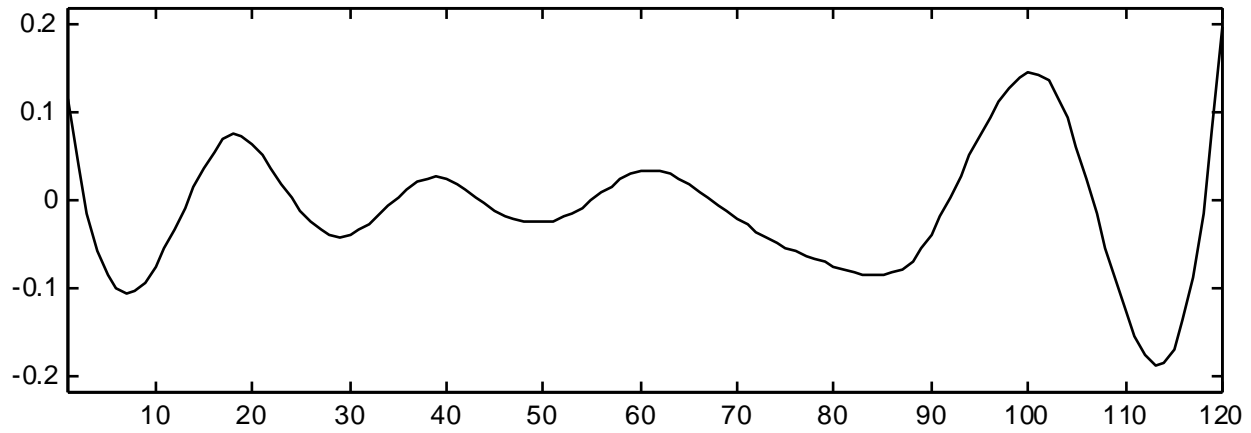
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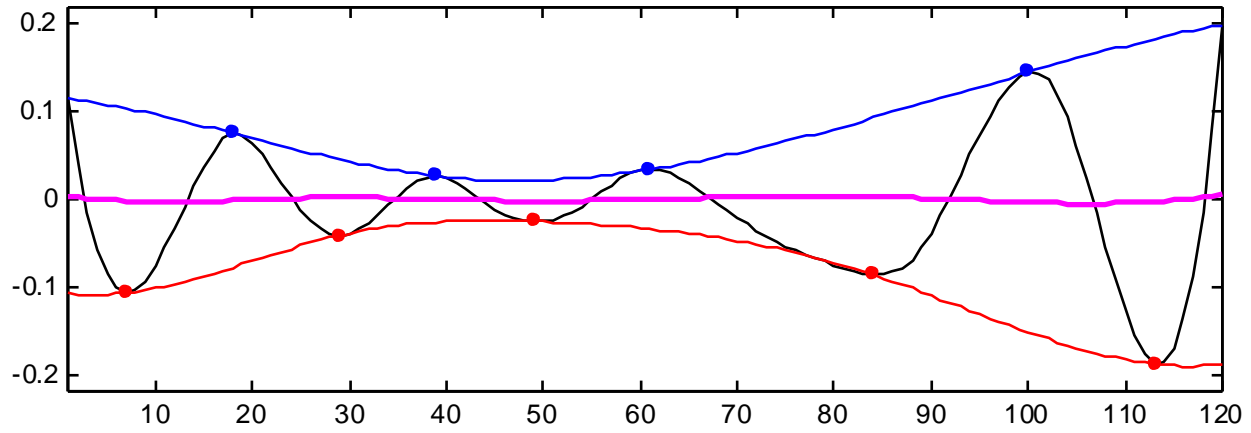
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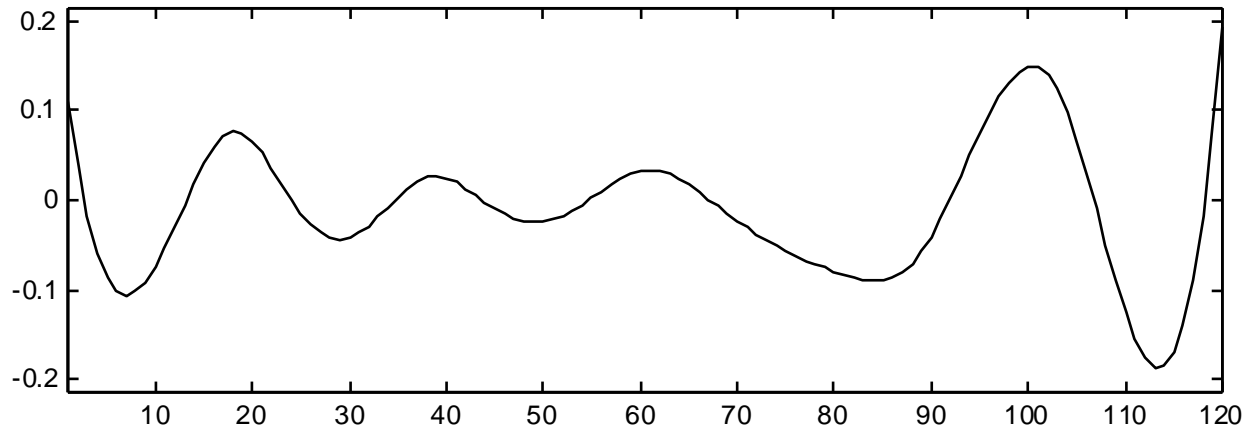
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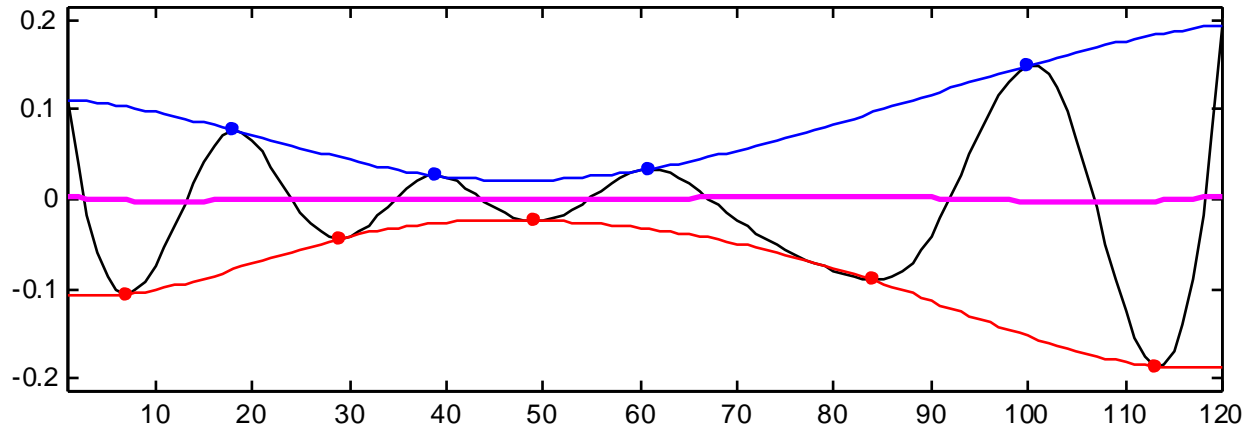
IMF 3; iteration 8



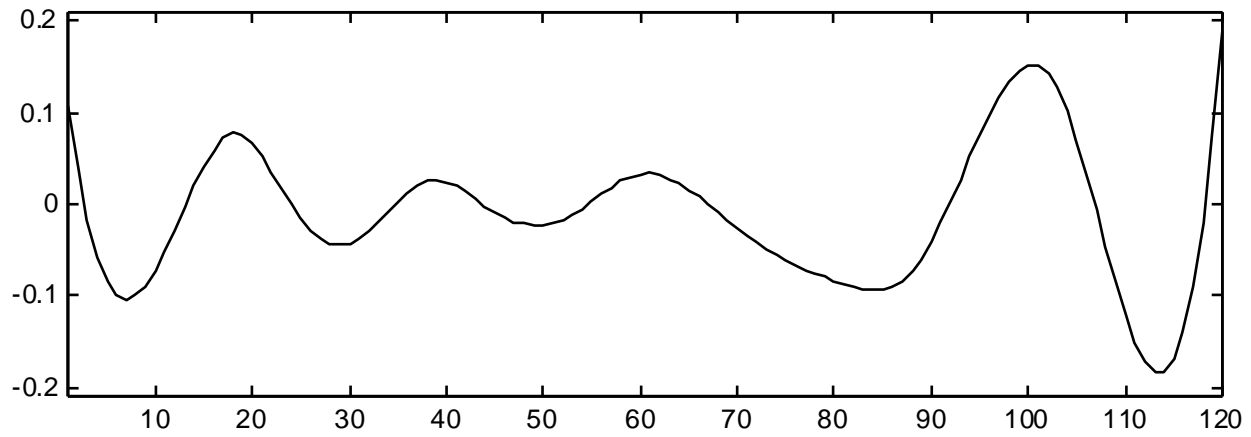
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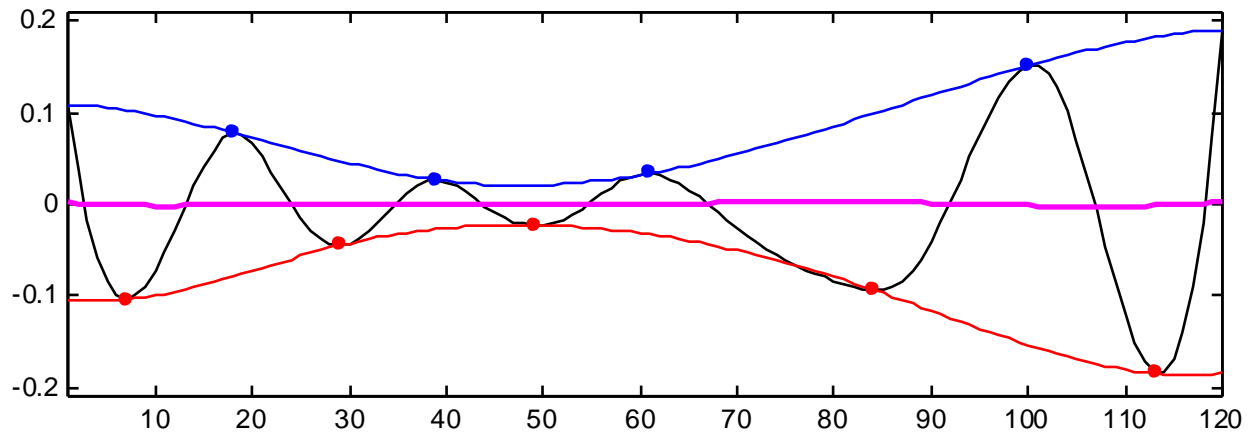
IMF 3; iteration 9



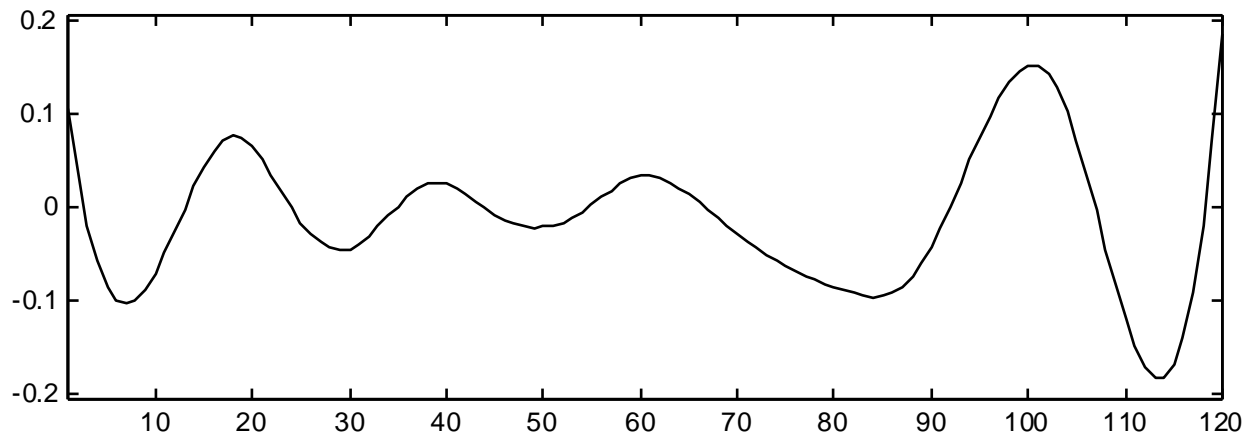
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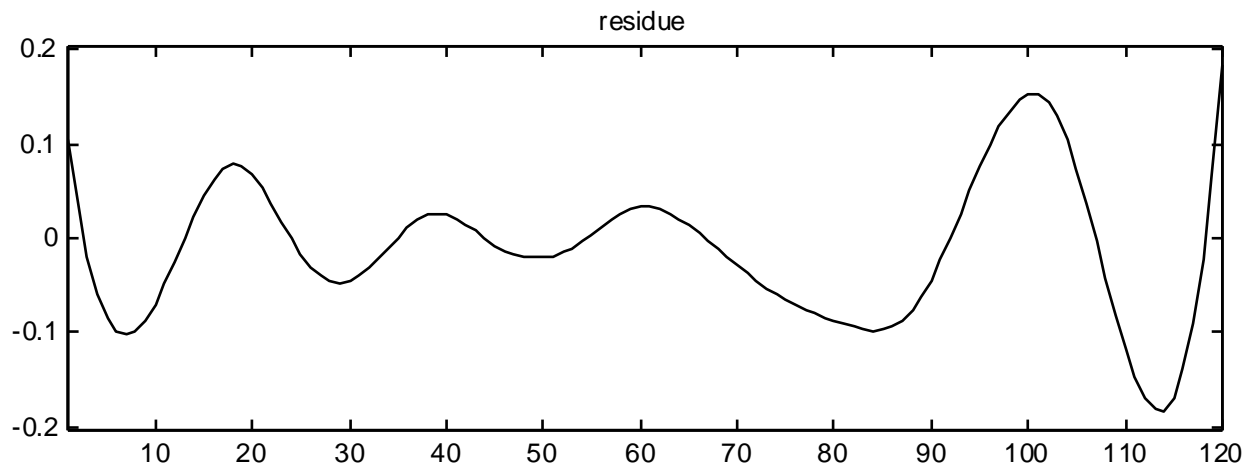
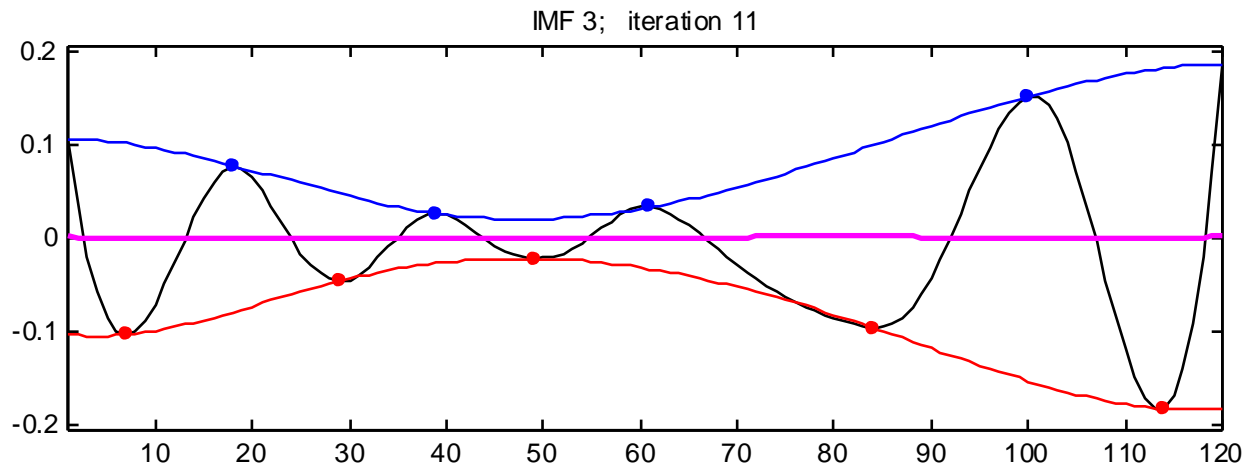


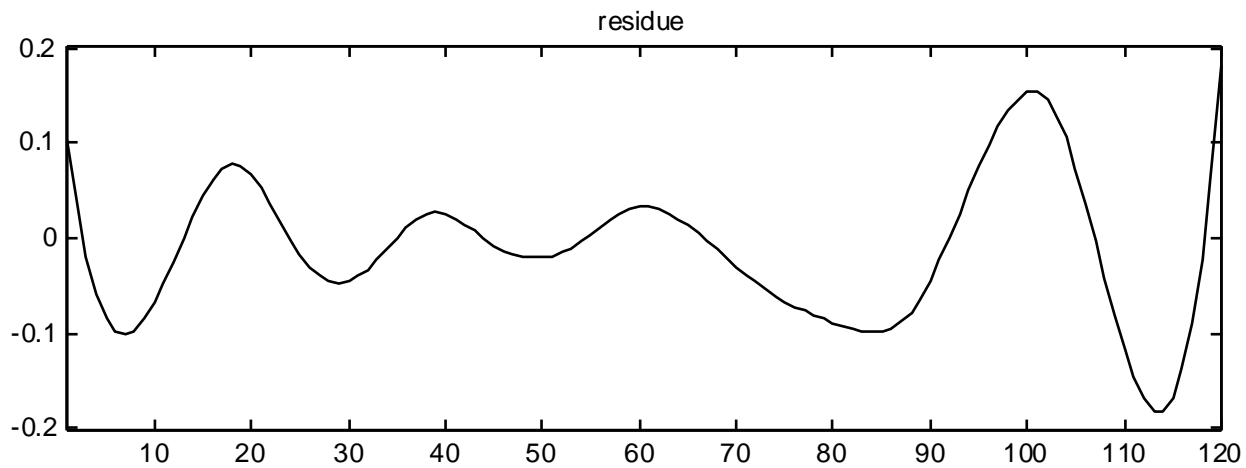
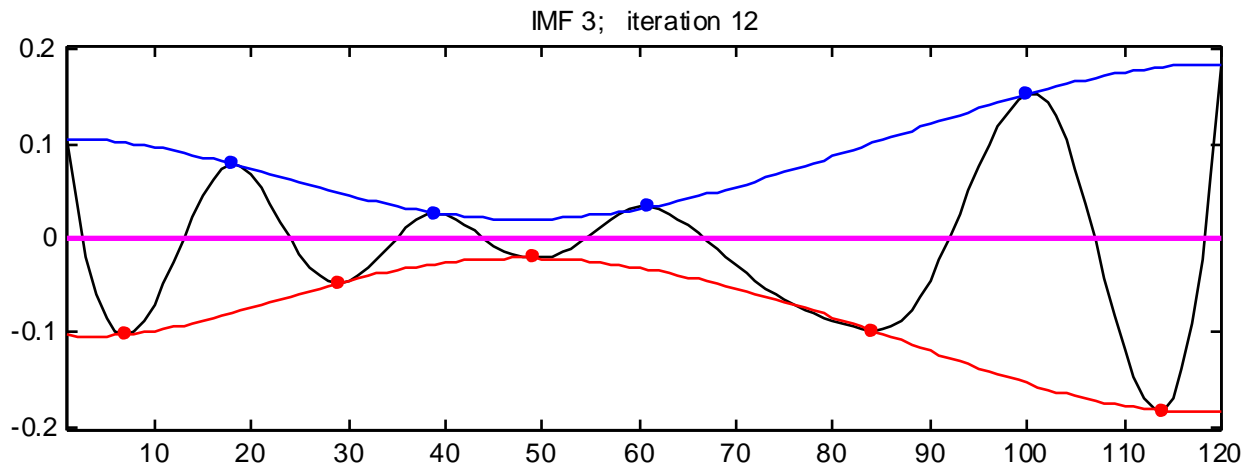
IMF 3; iteration 10



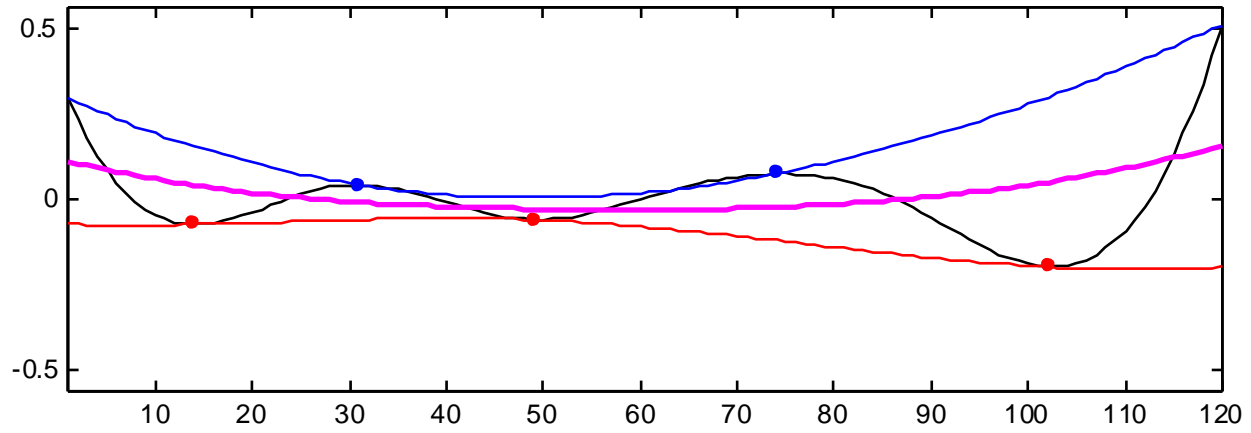
residue



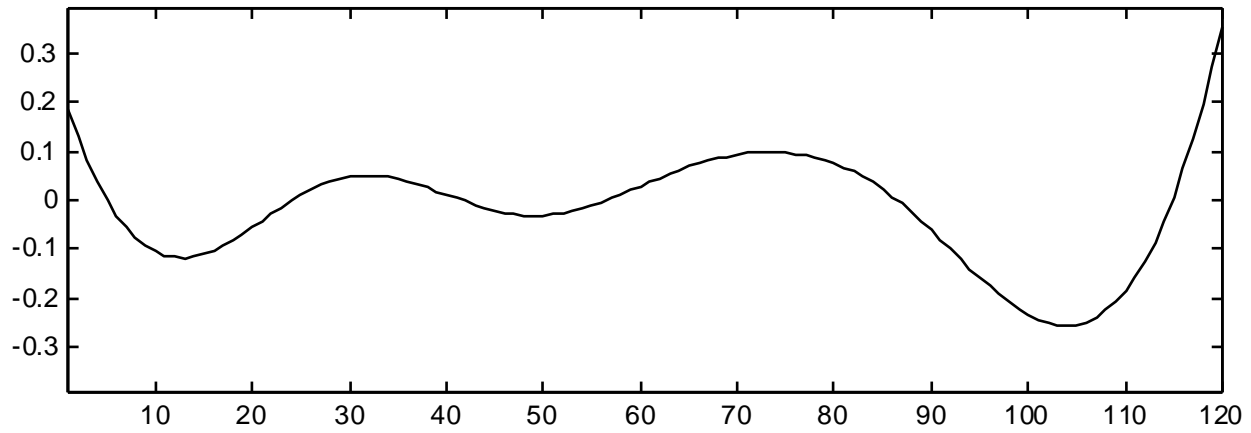




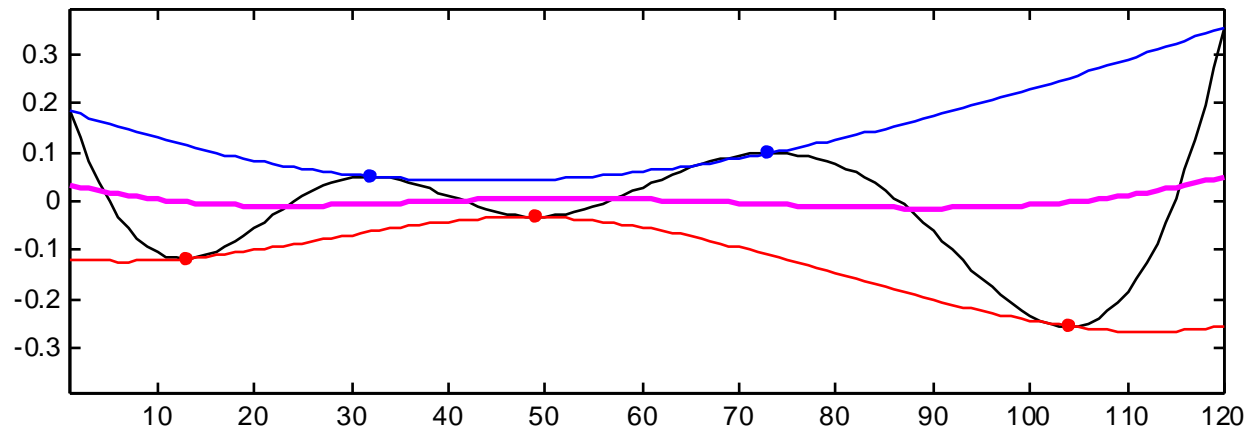
IMF 4; iteration 0



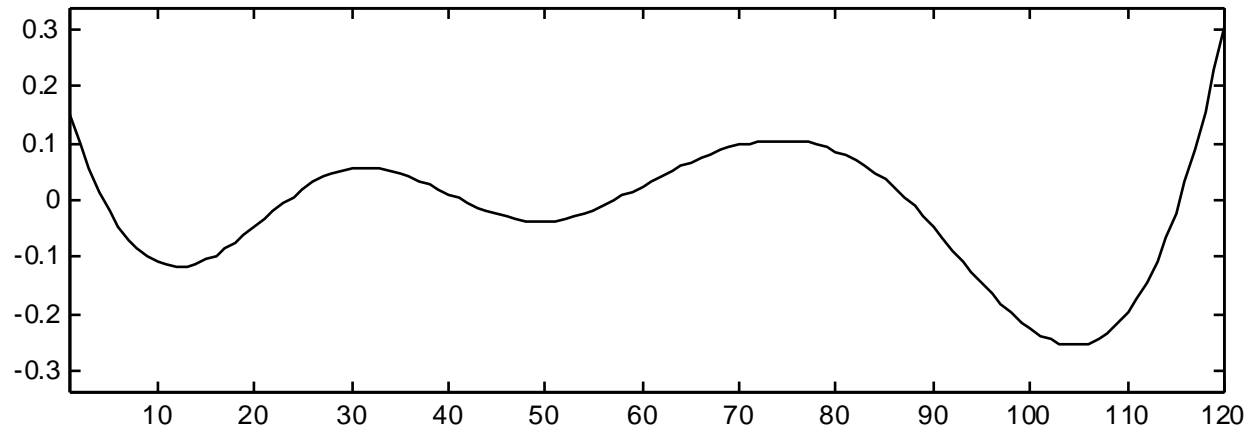
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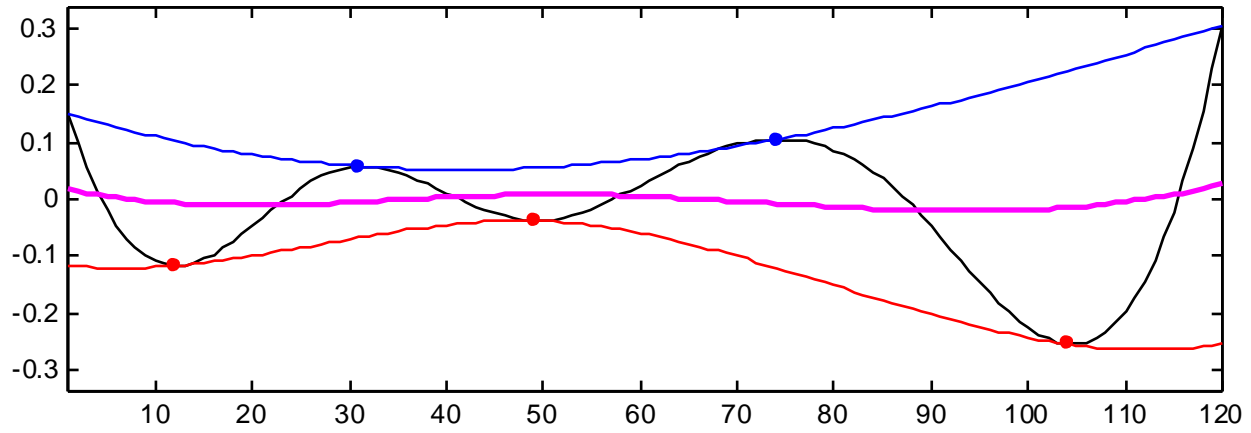
IMF 4; iteration 1



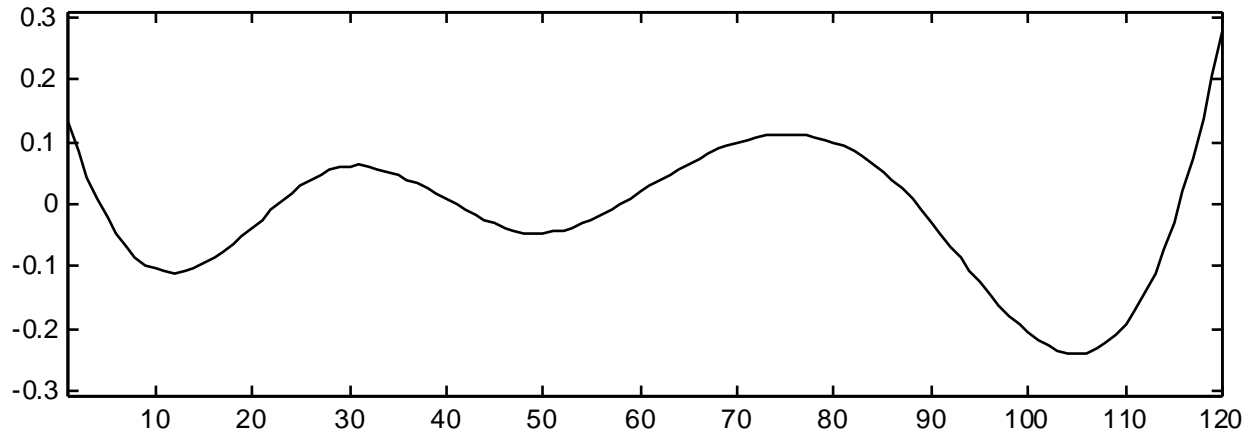
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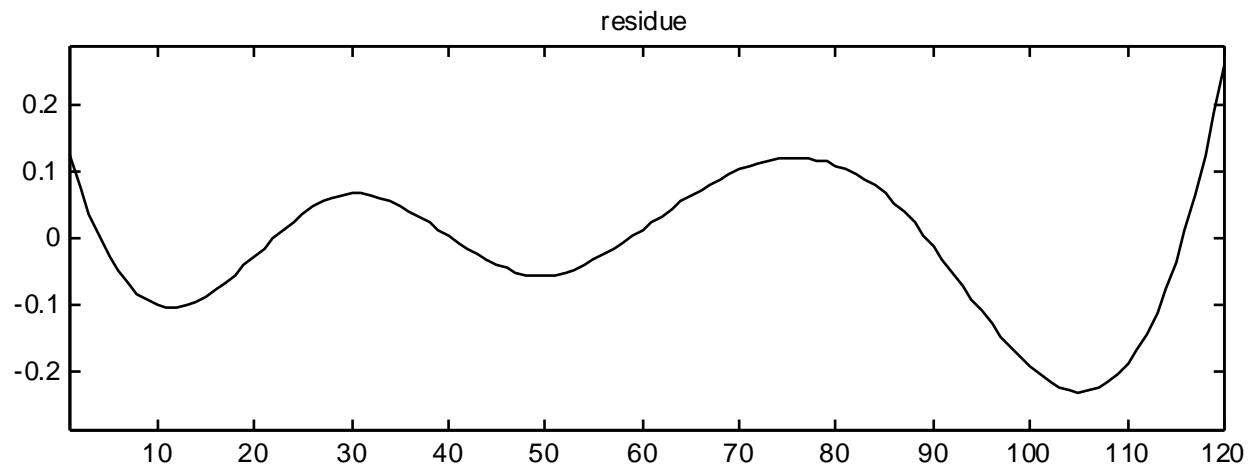
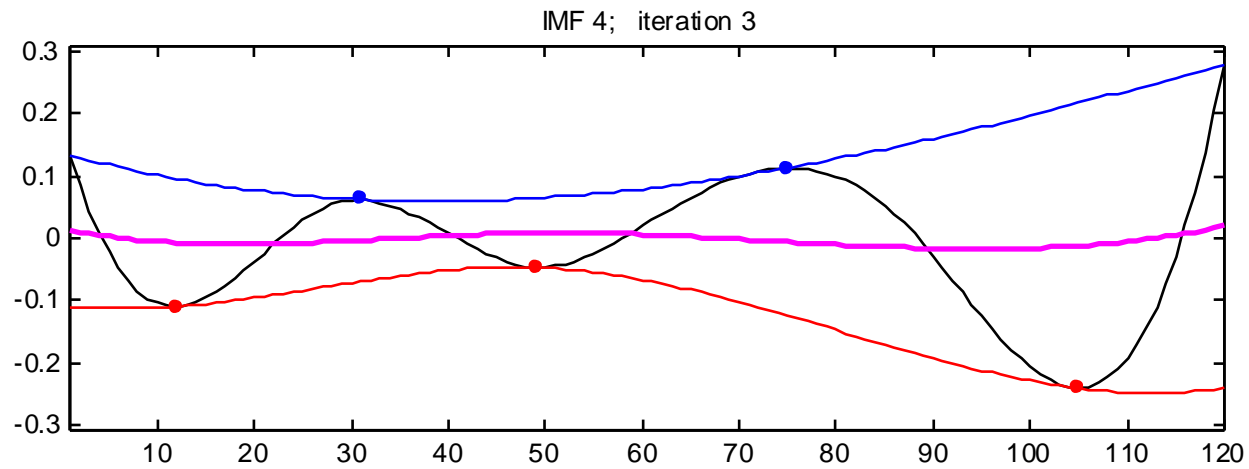


IMF 4; iteration 2

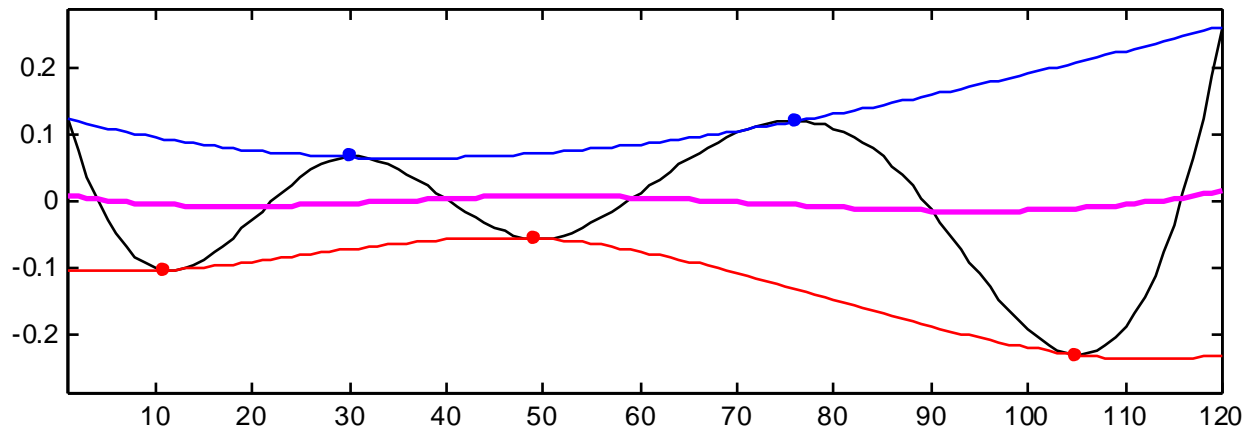


residue

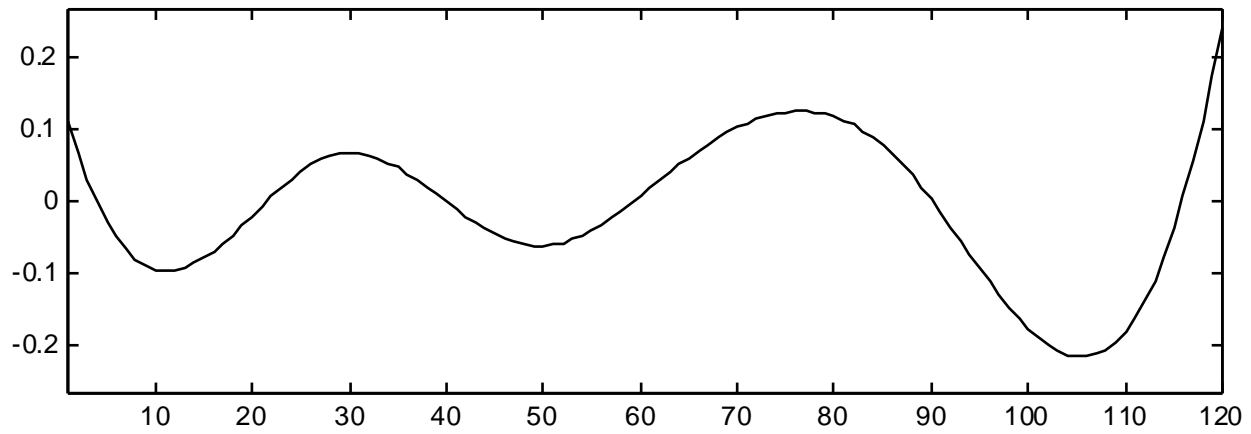




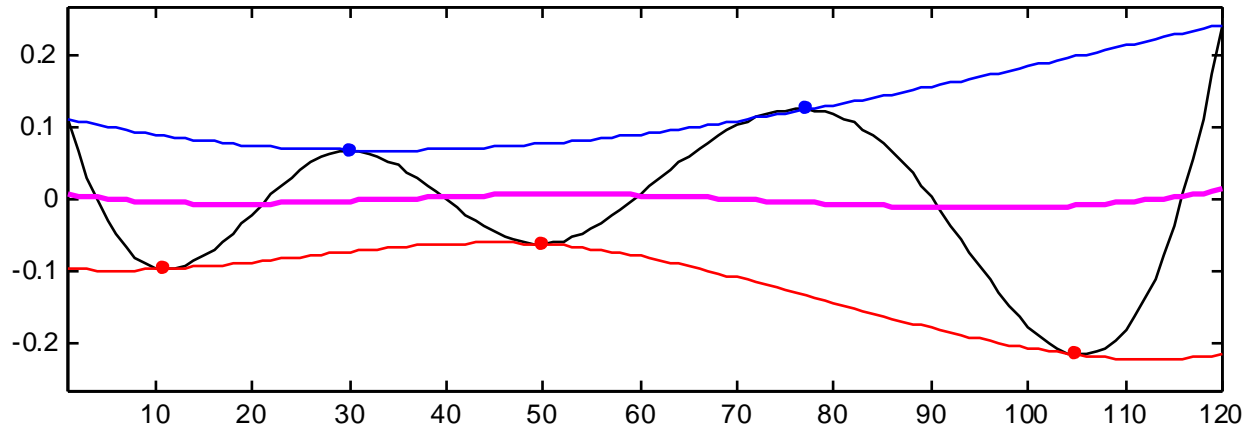
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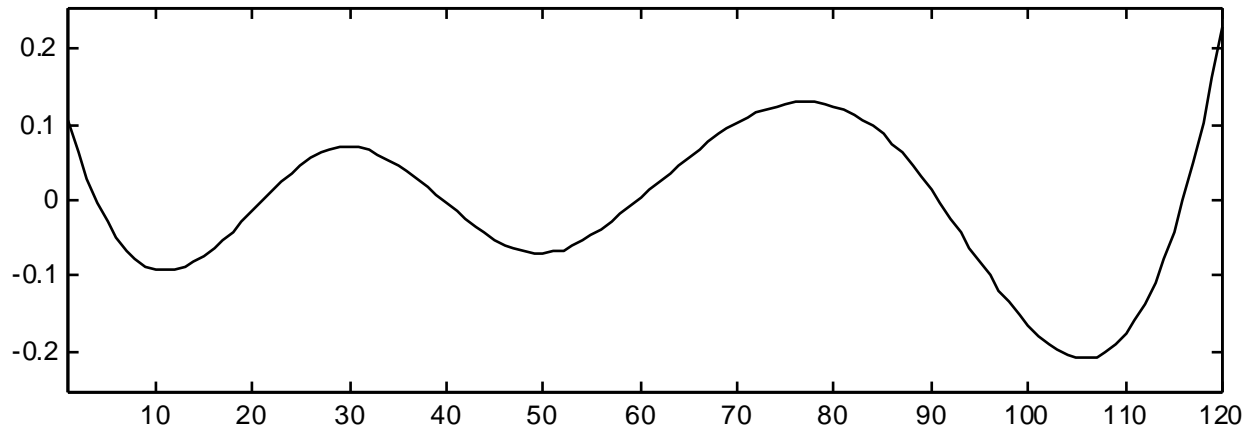
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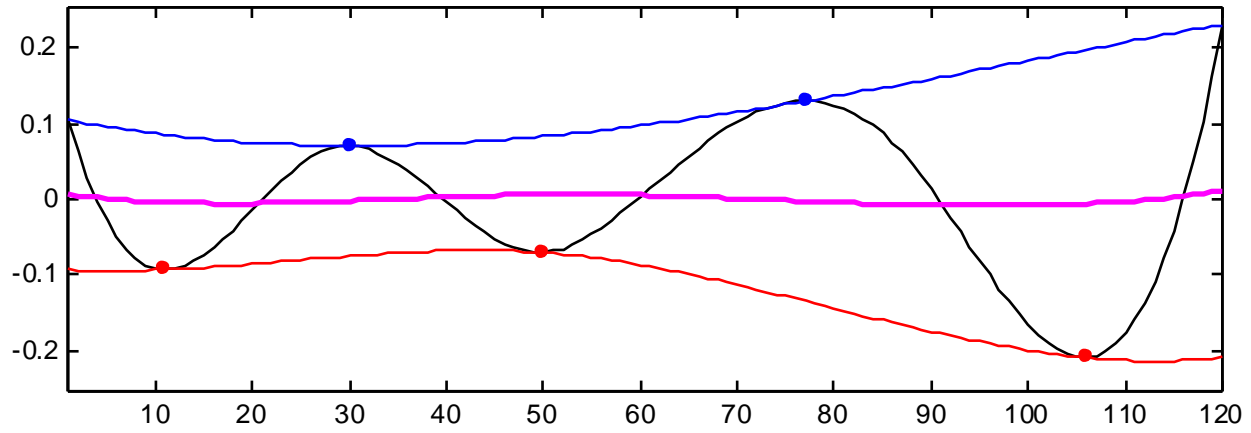
IMF 4; iteration 5



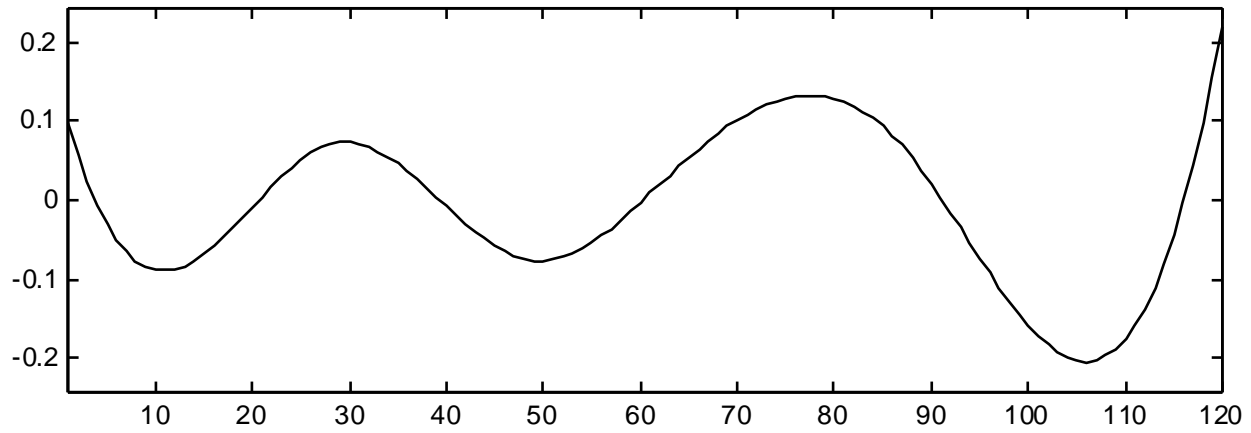
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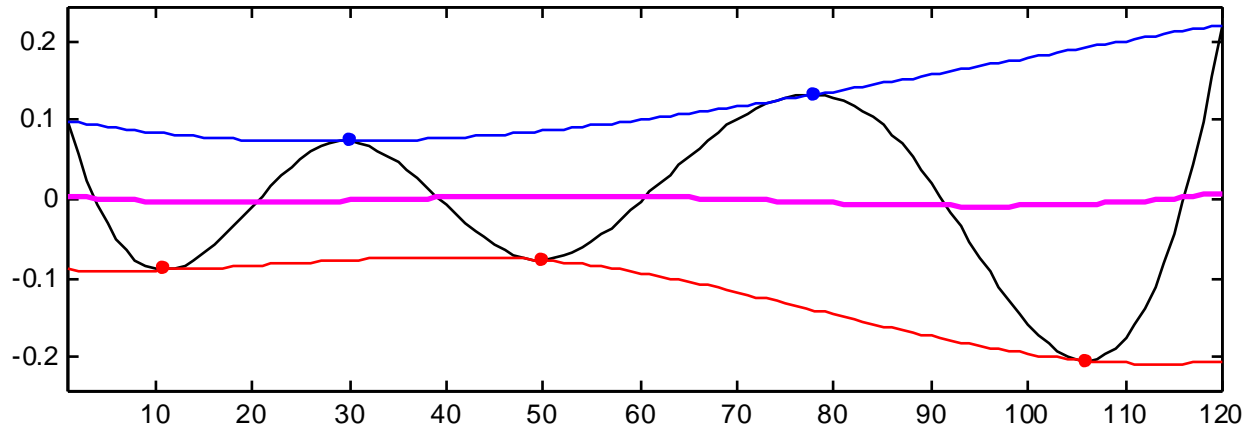
IMF 4; iteration 6



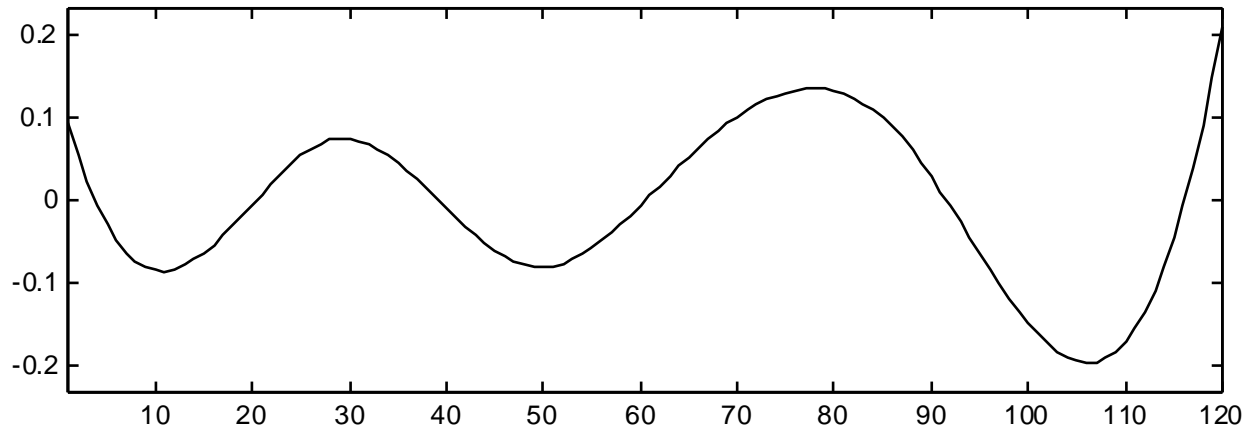
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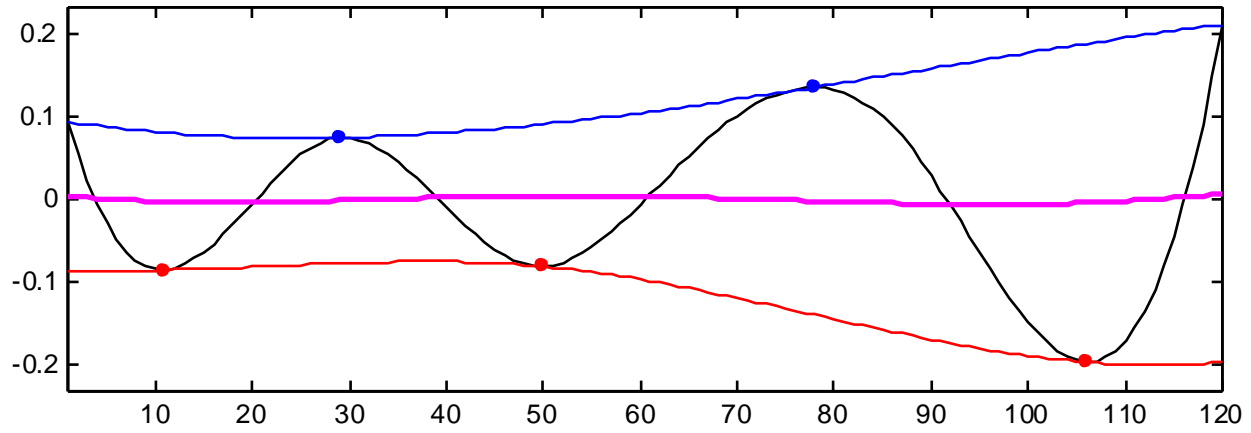
IMF 4; iteration 7



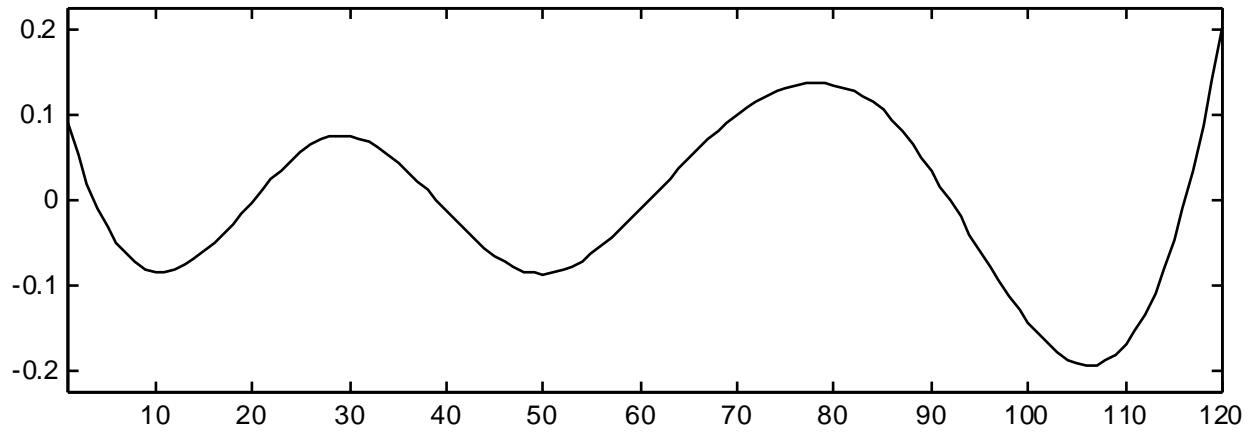
residue



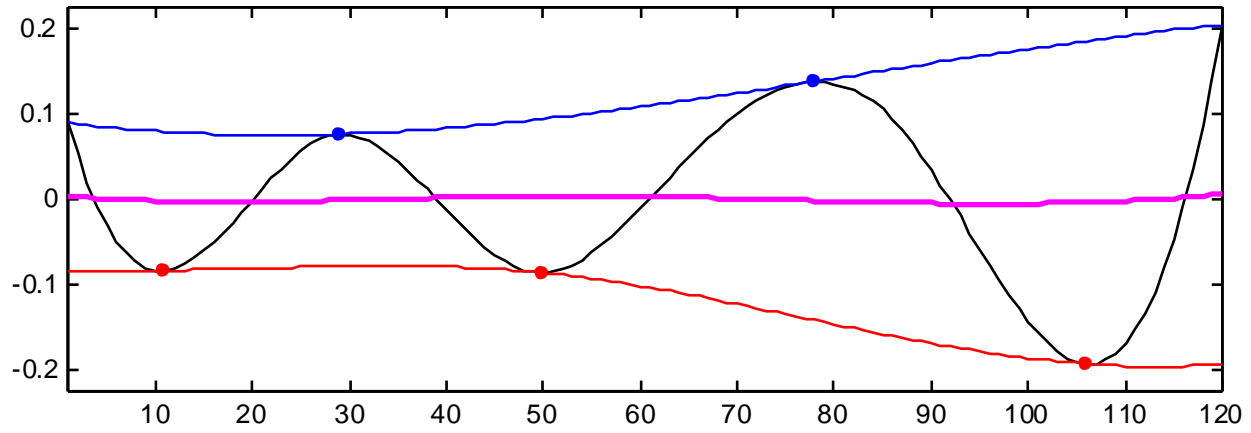
IMF 4; iteration 8



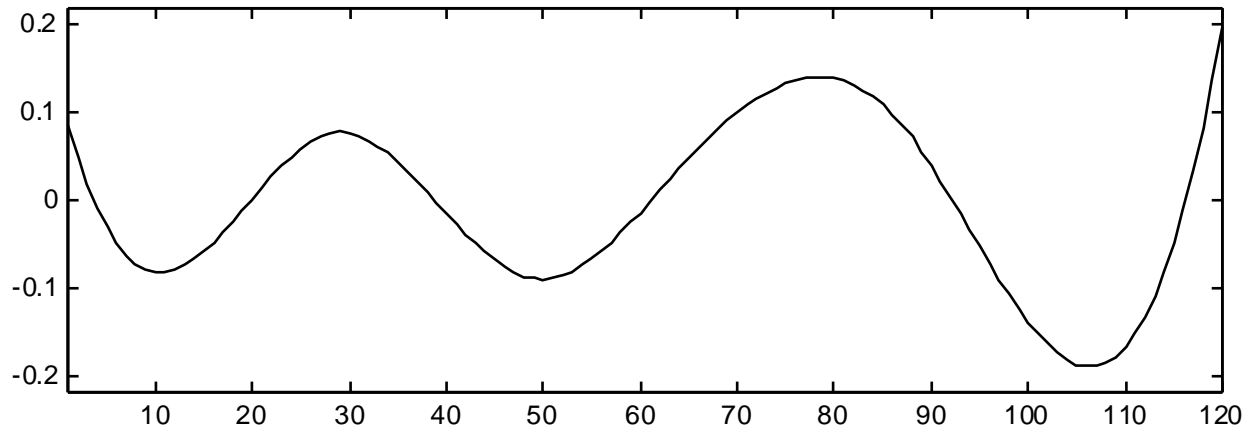
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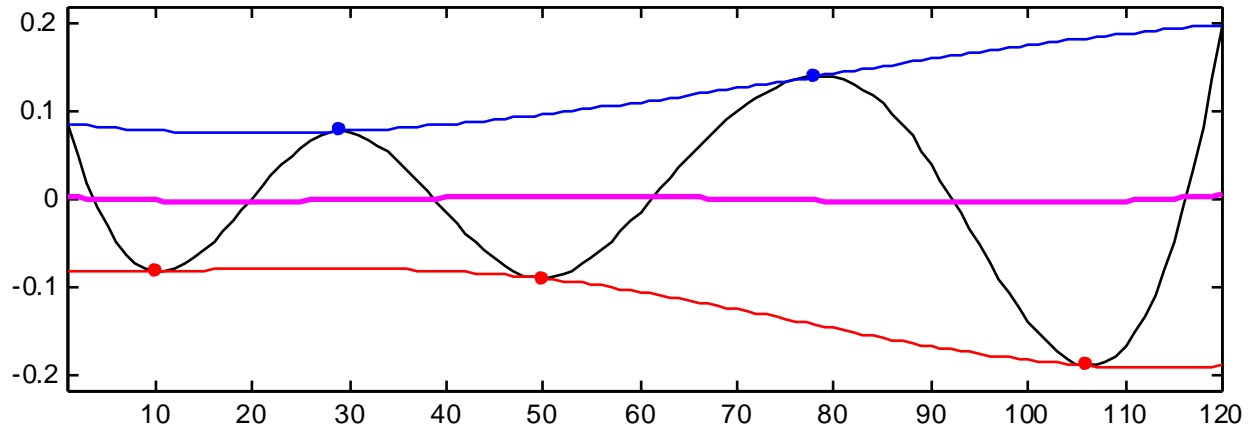
IMF 4; iteration 9



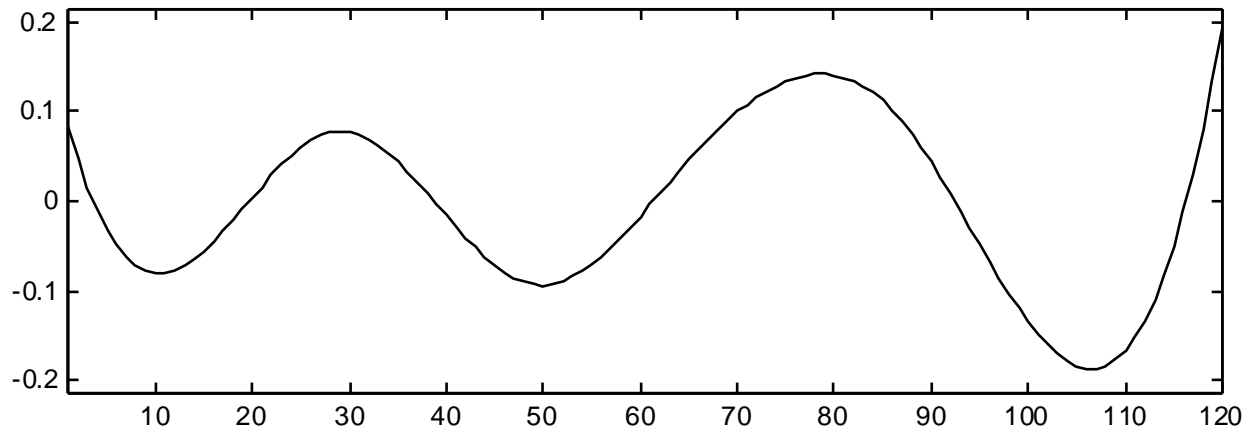
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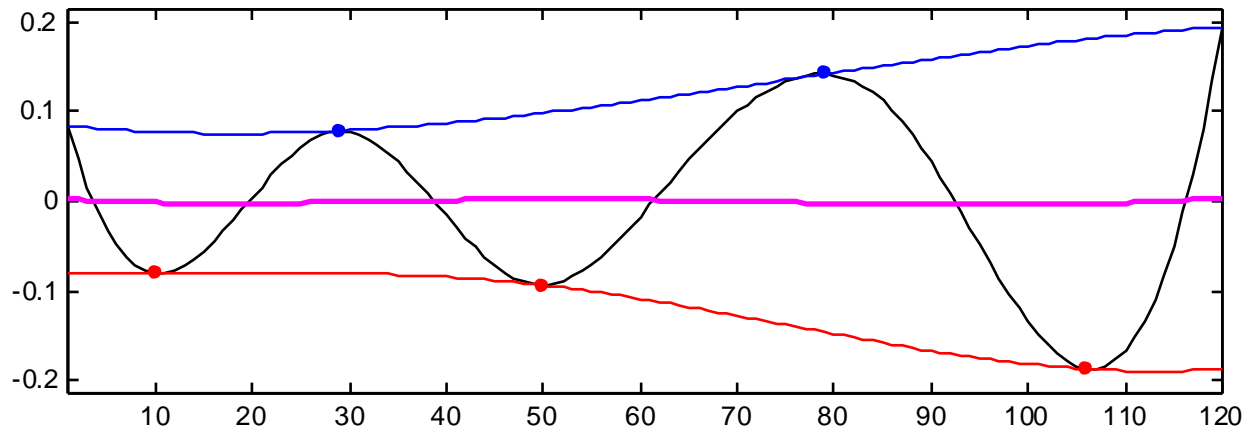
IMF 4; iteration 10



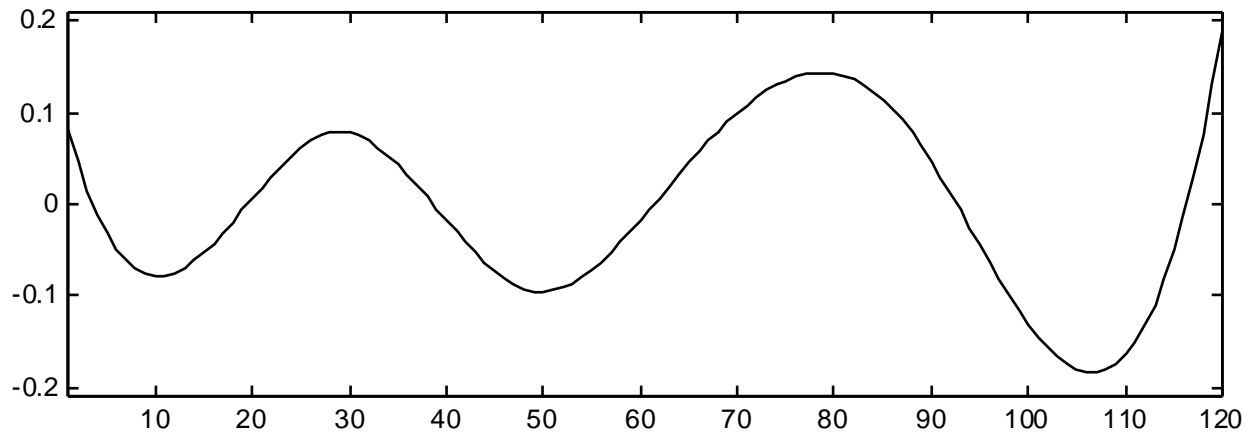
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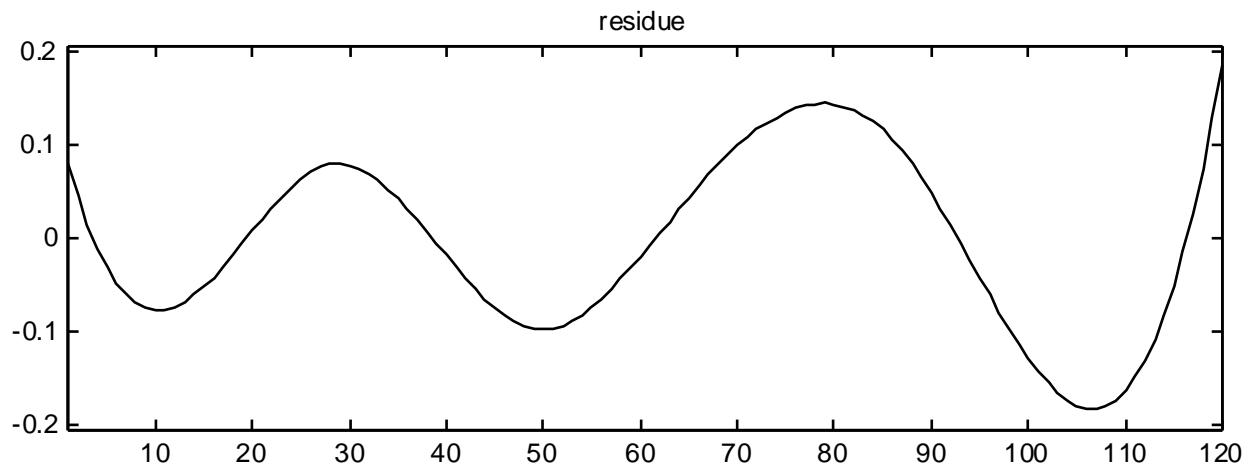
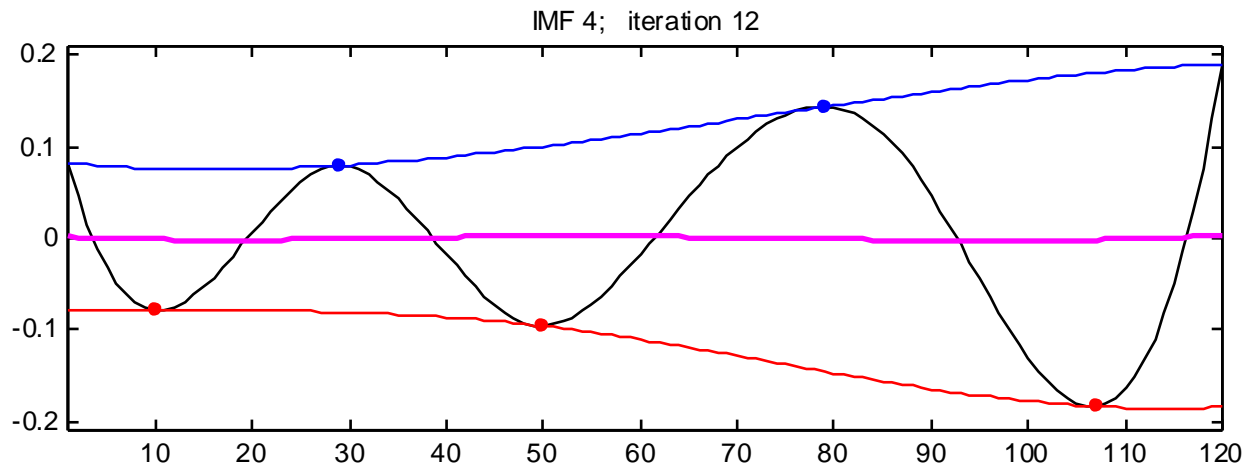


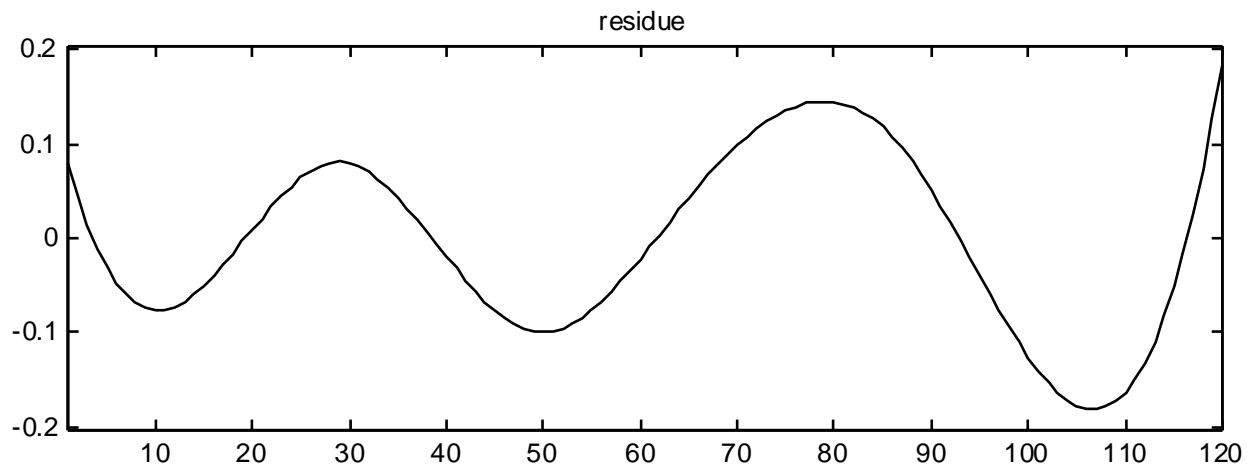
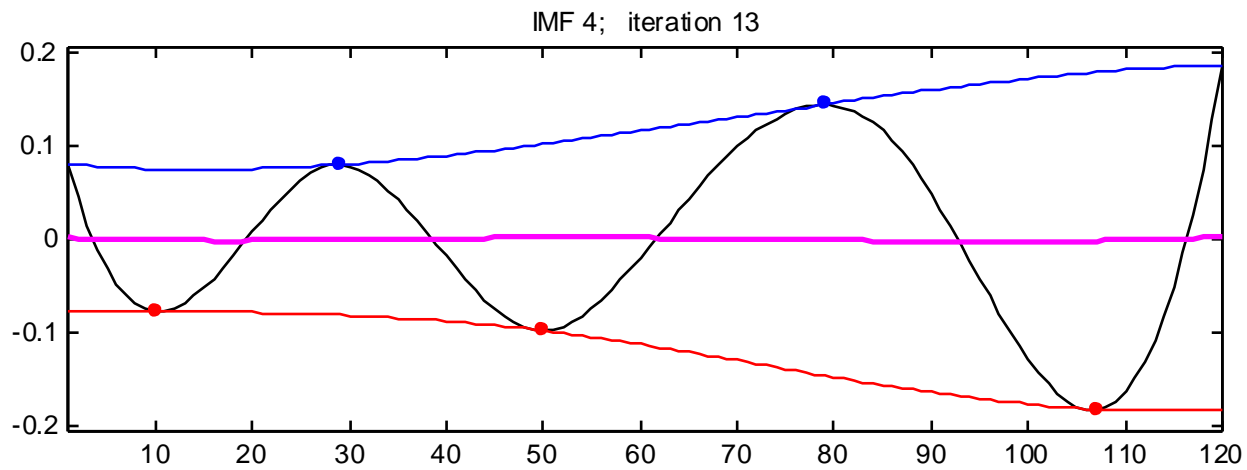
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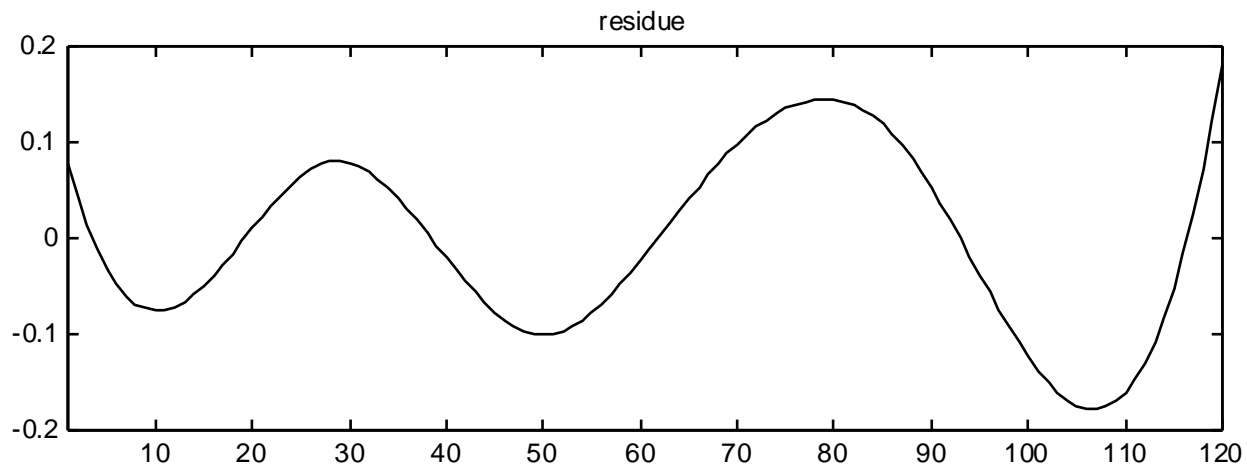
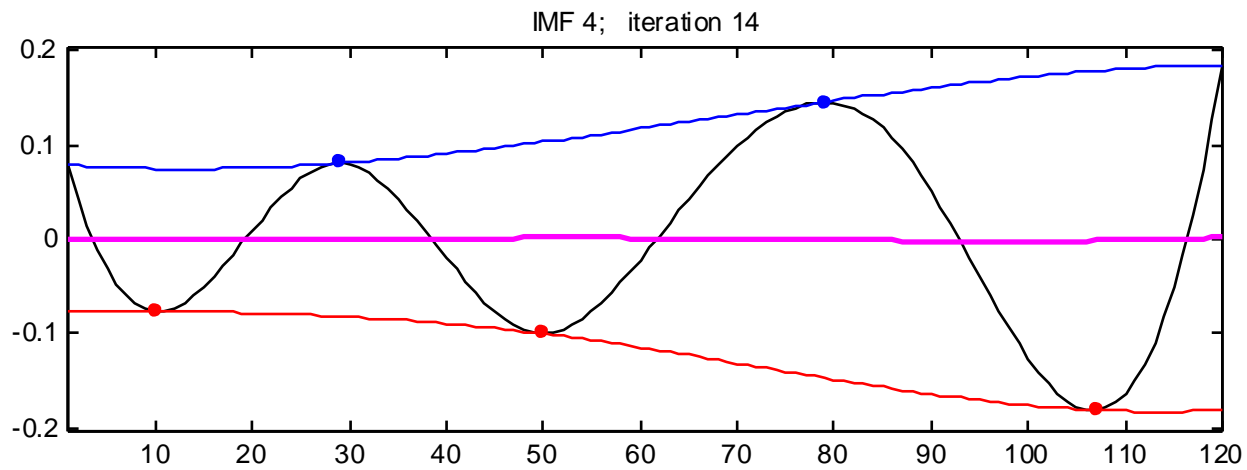


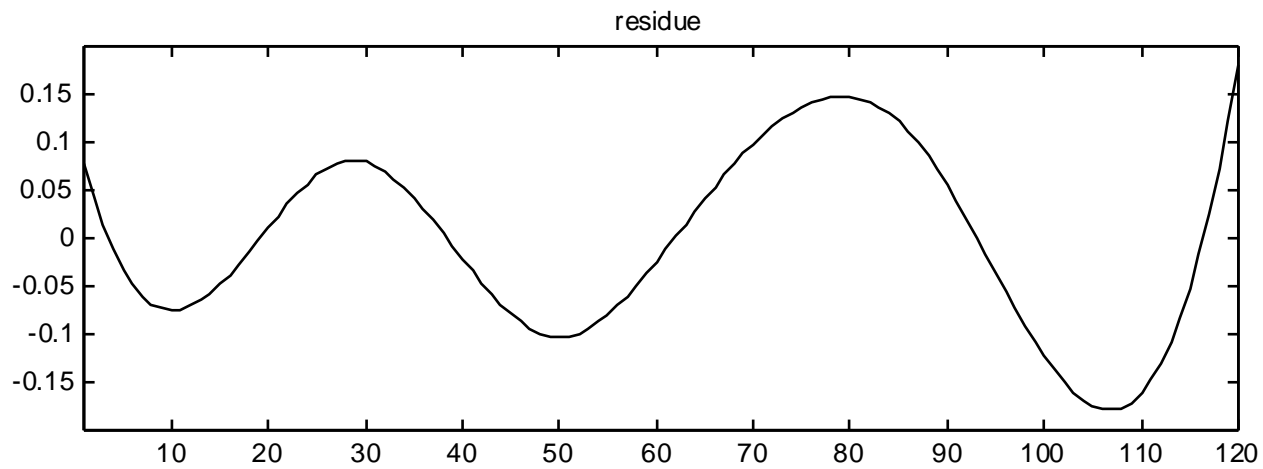
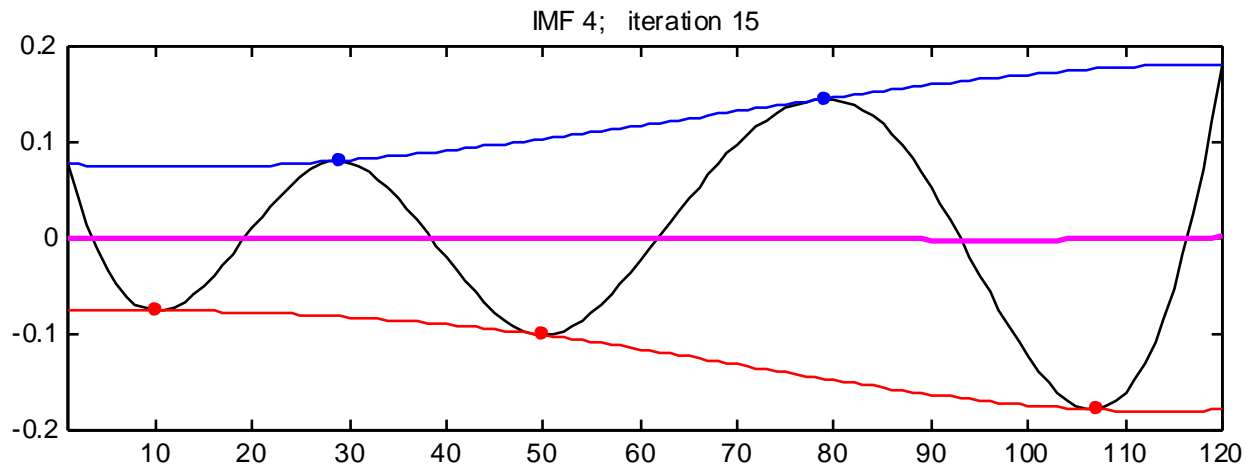
residue



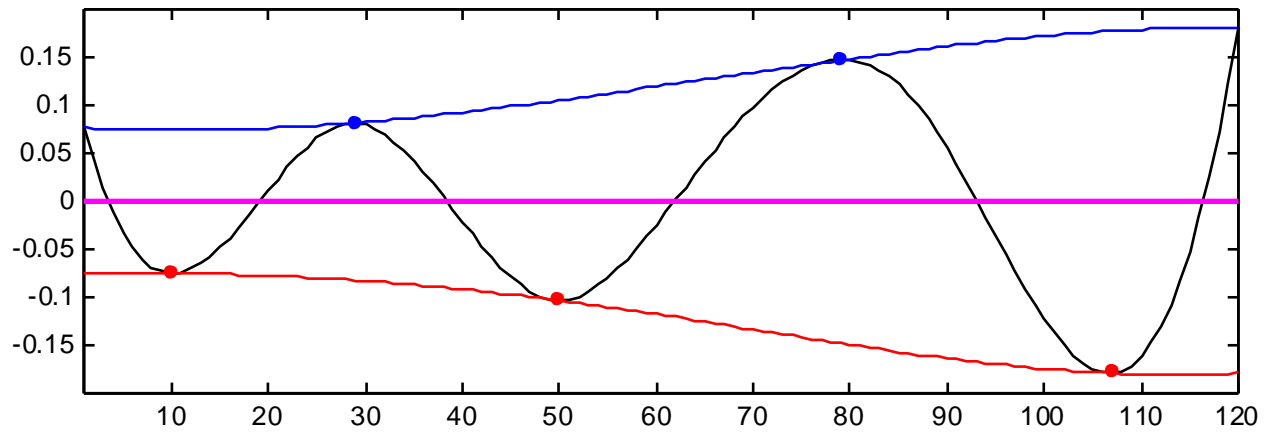




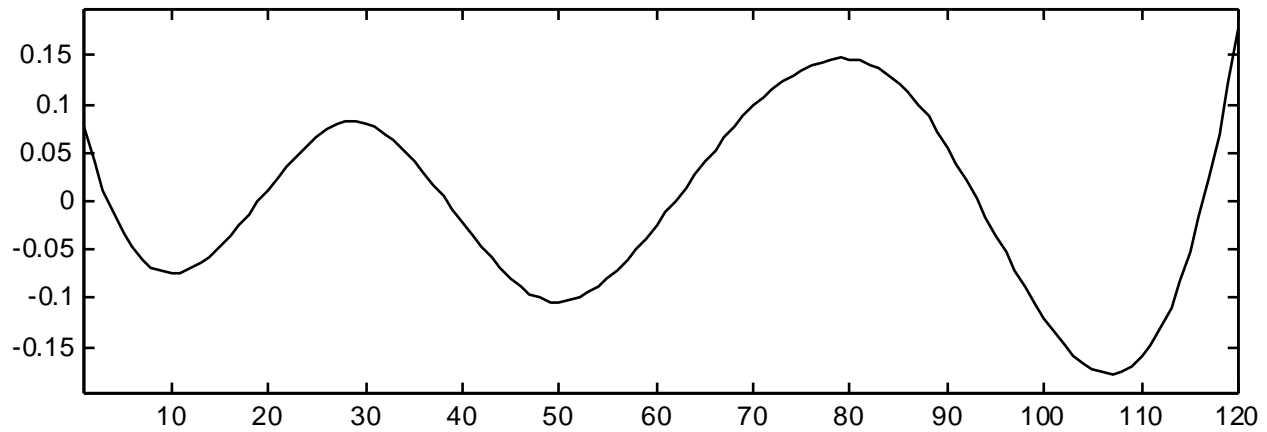




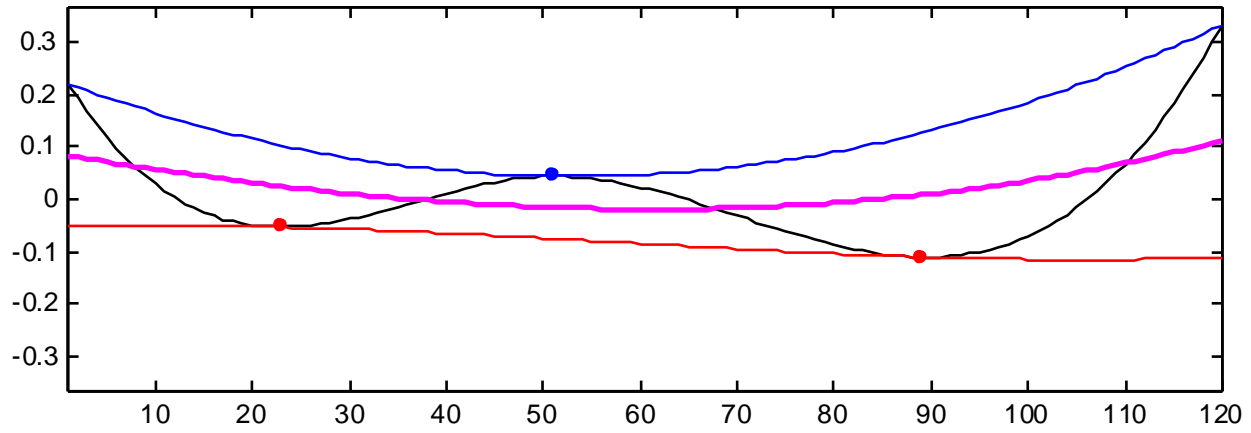
IMF 4; iteration 16



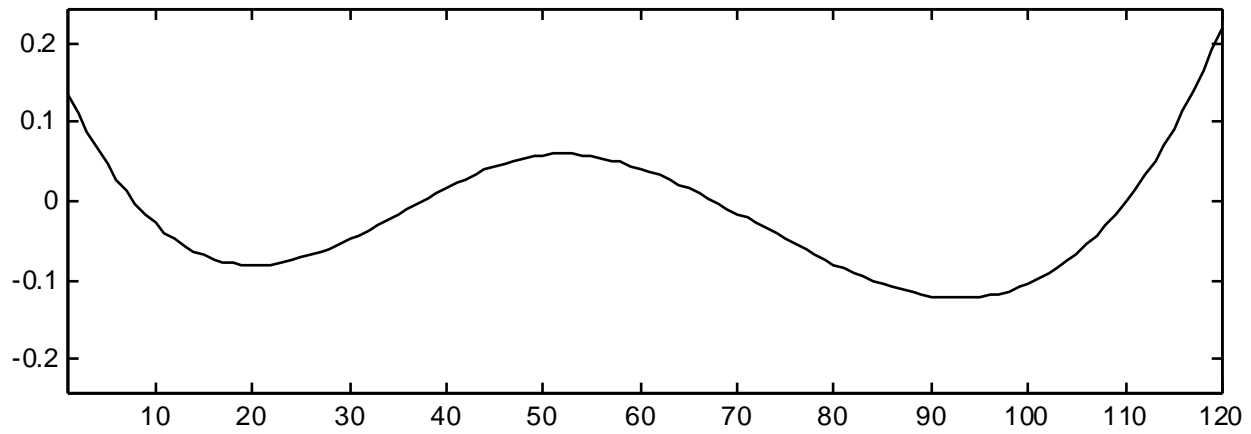
residue



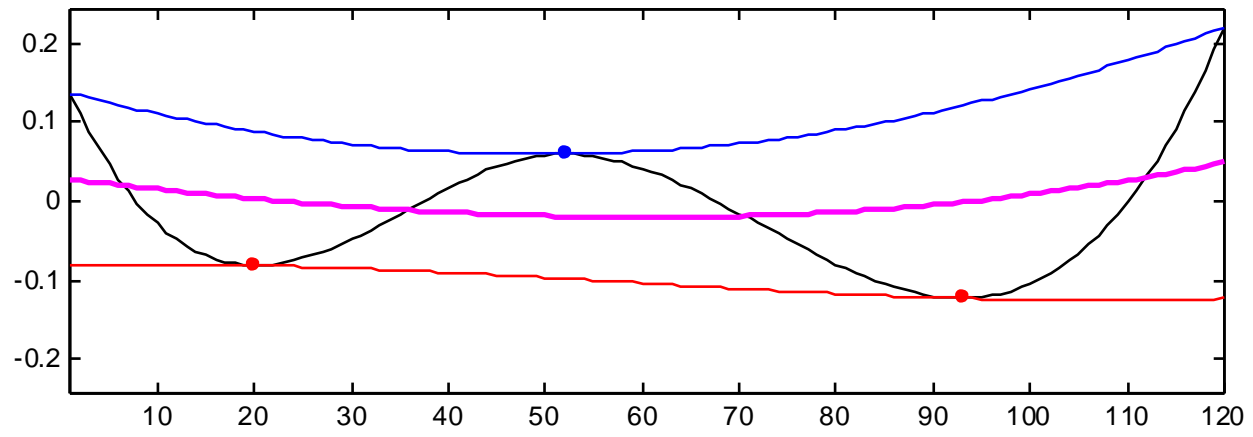
IMF 5; iteration 0



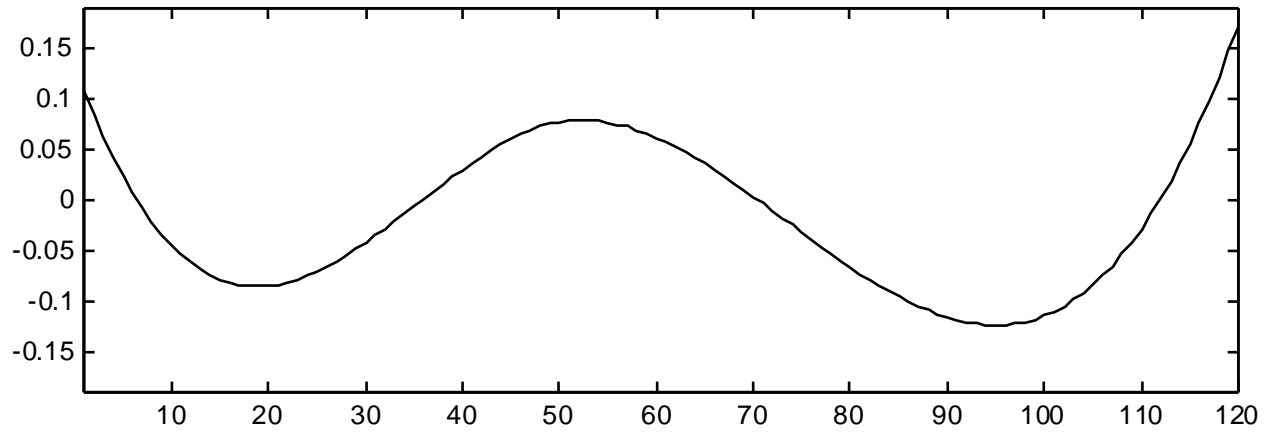
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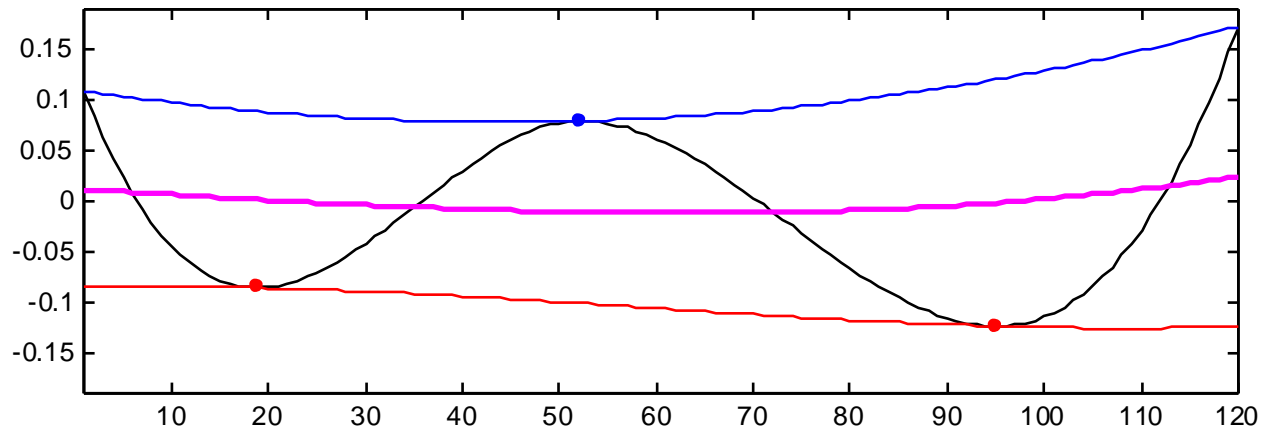
IMF 5; iteration 1



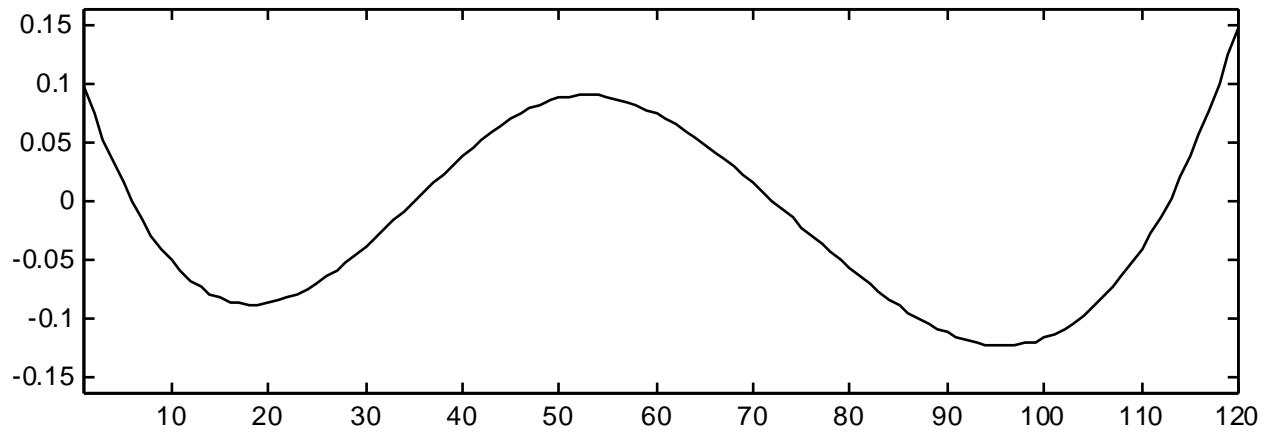
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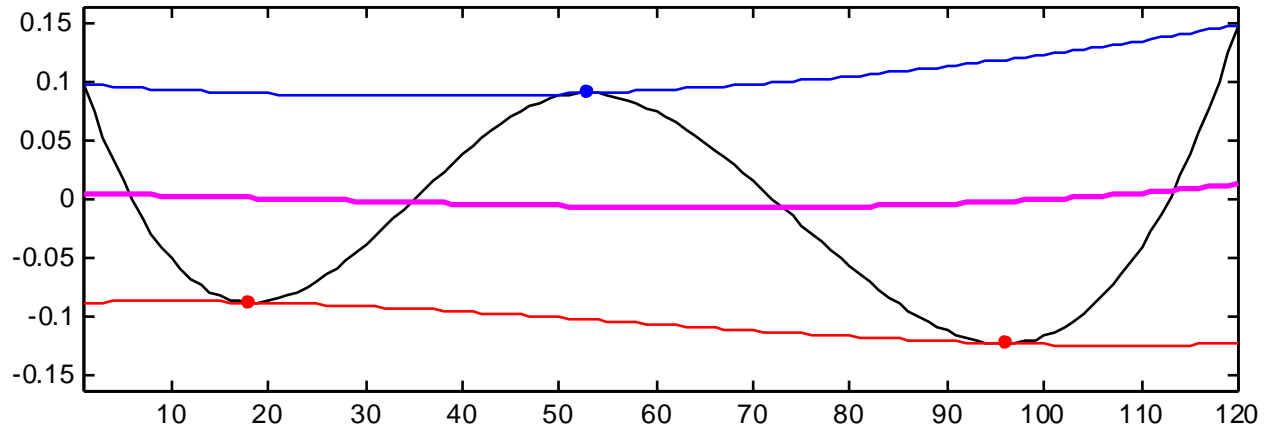
IMF 5; iteration 2



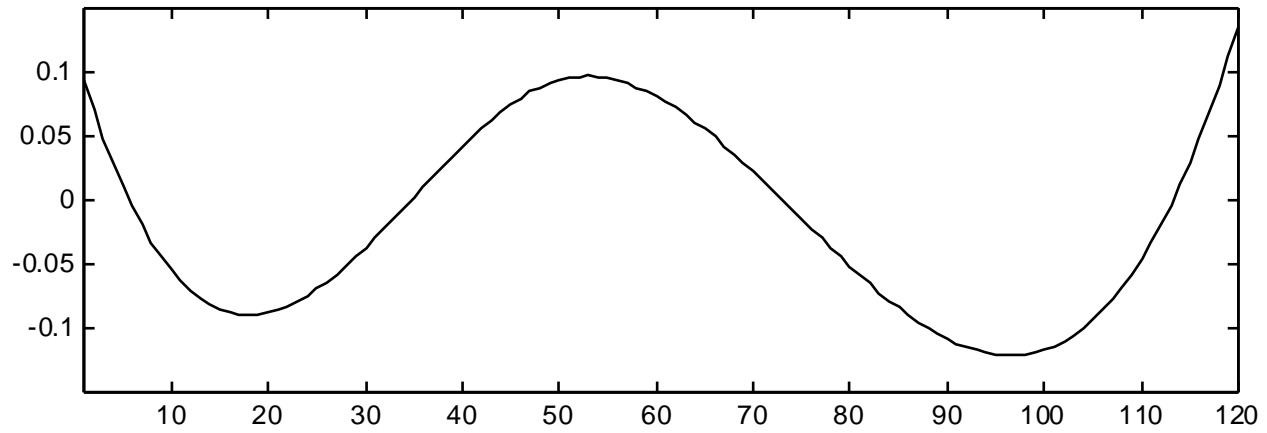
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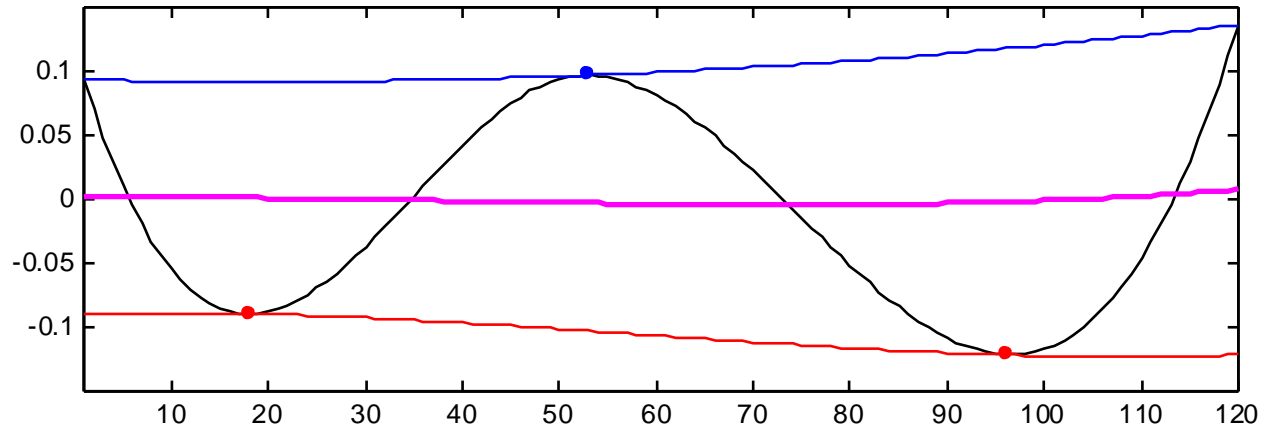
IMF 5; iteration 3



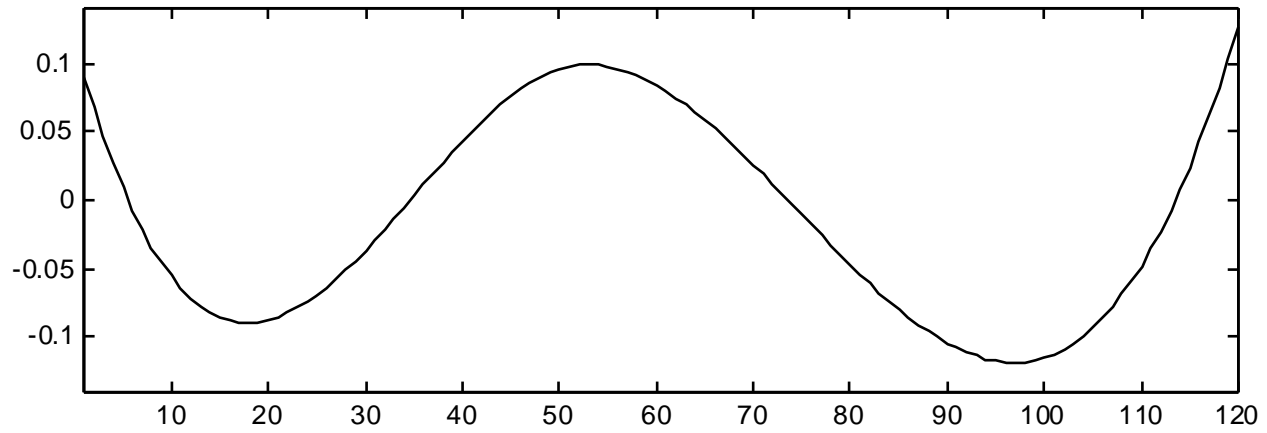
residue



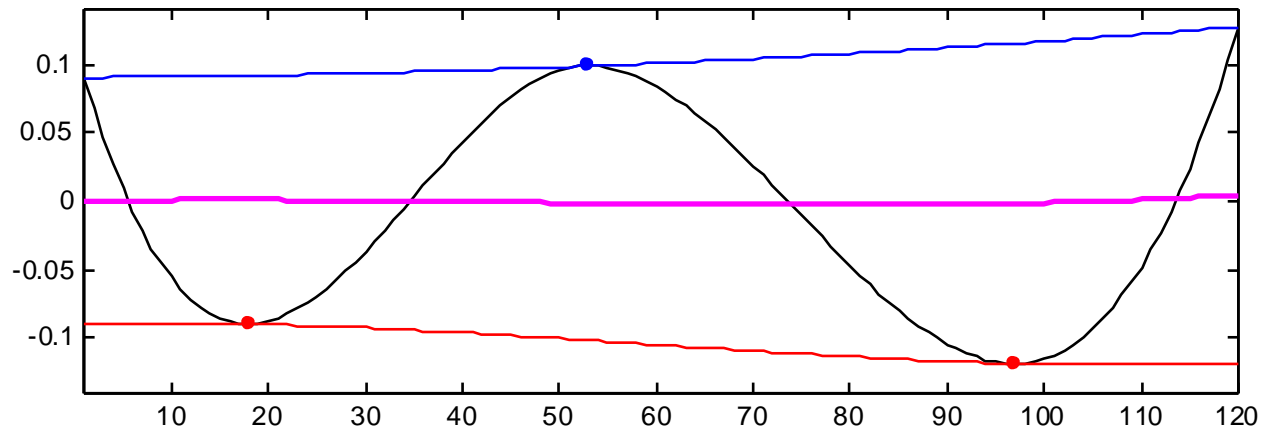
IMF 5; iteration 4



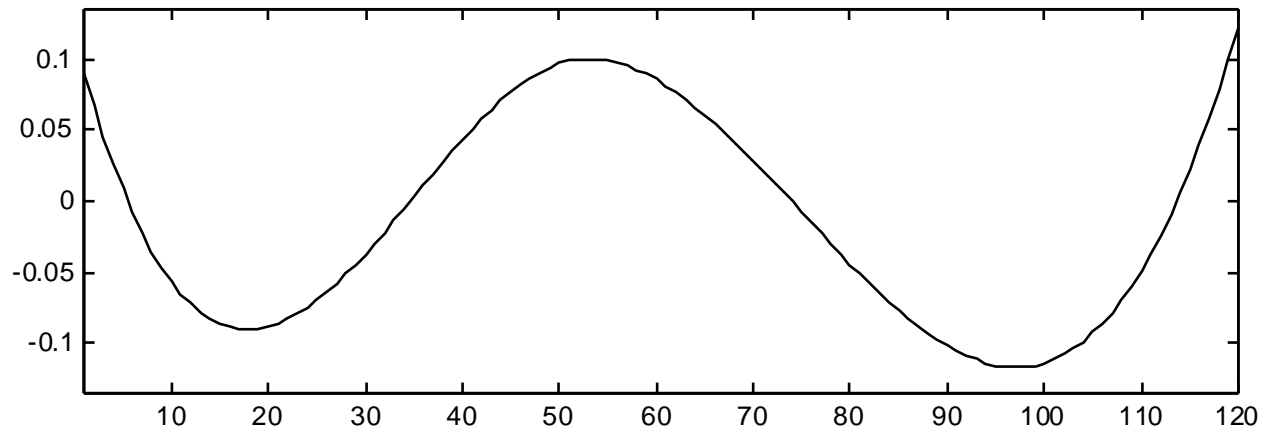
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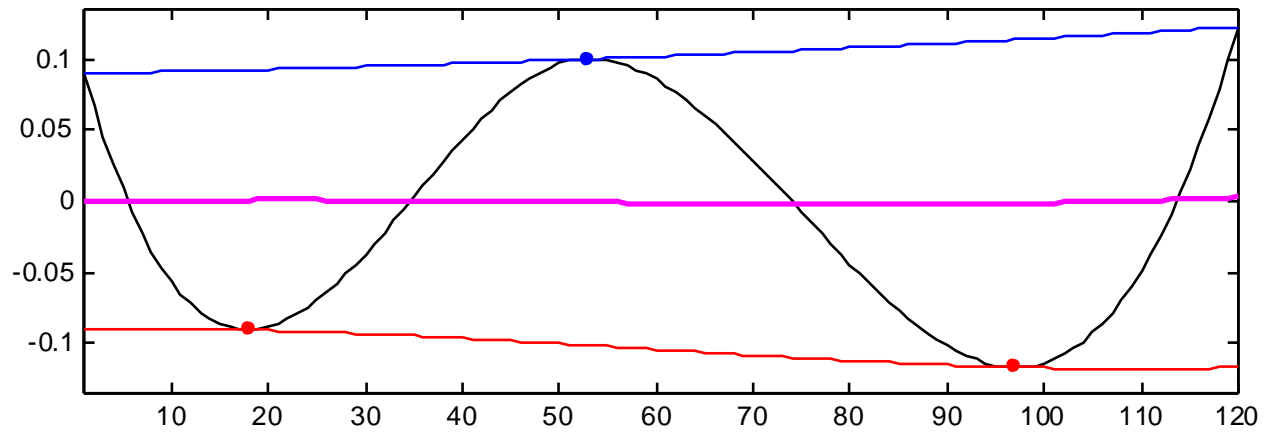
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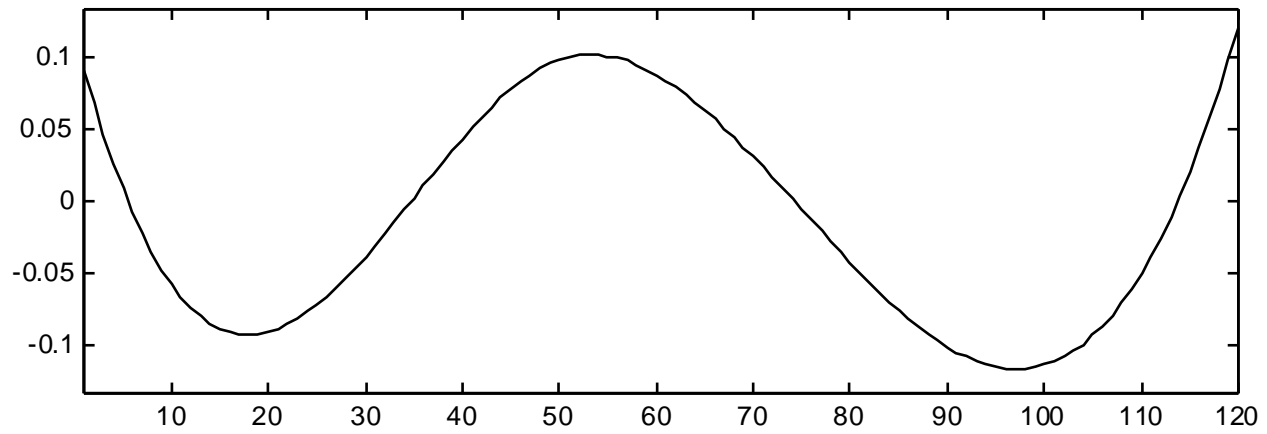
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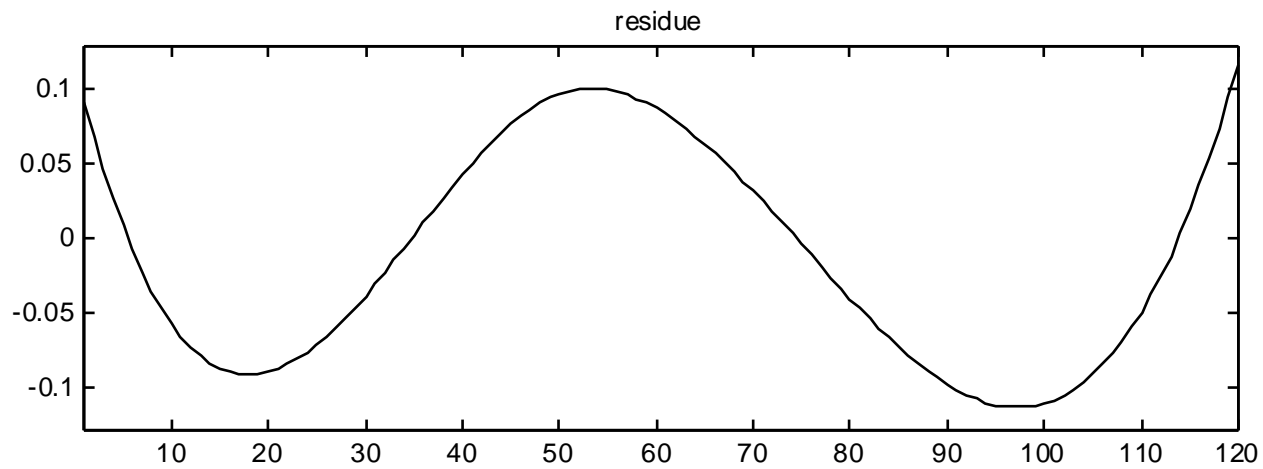
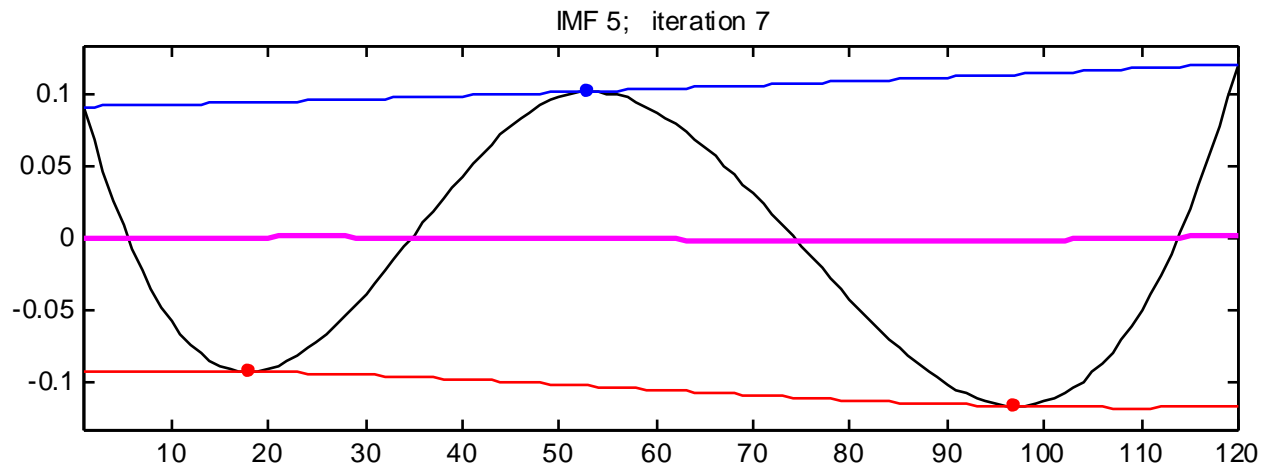


IMF 5; iteration 6

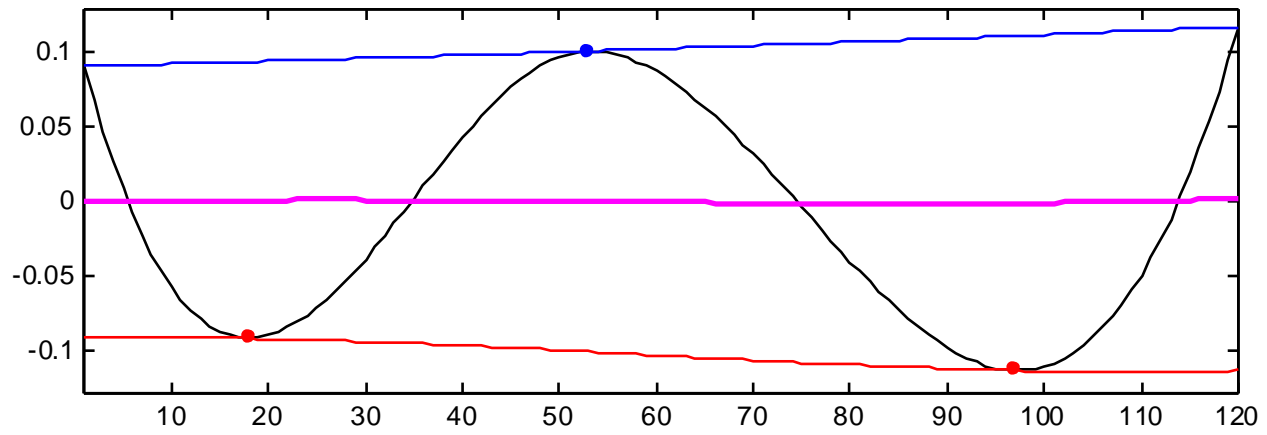


residue

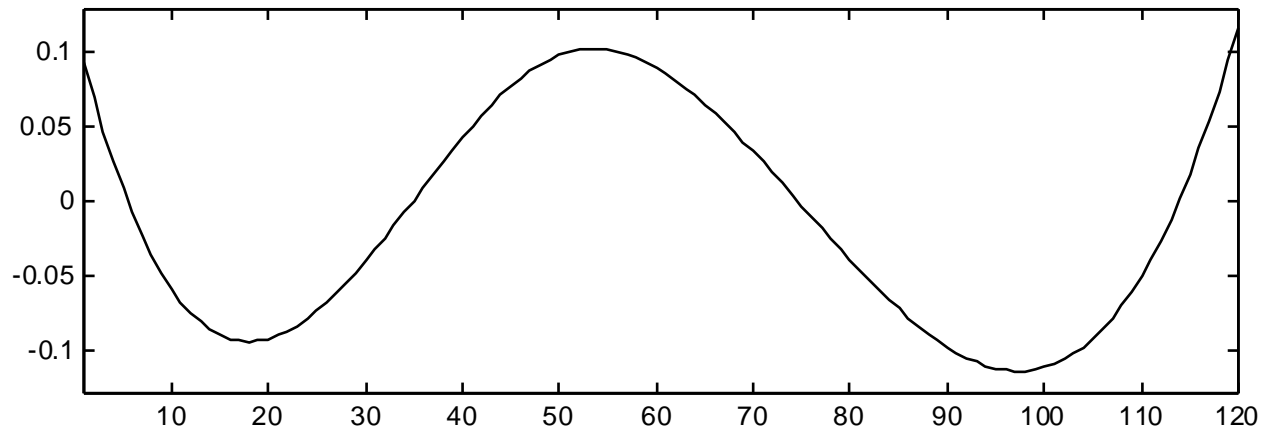




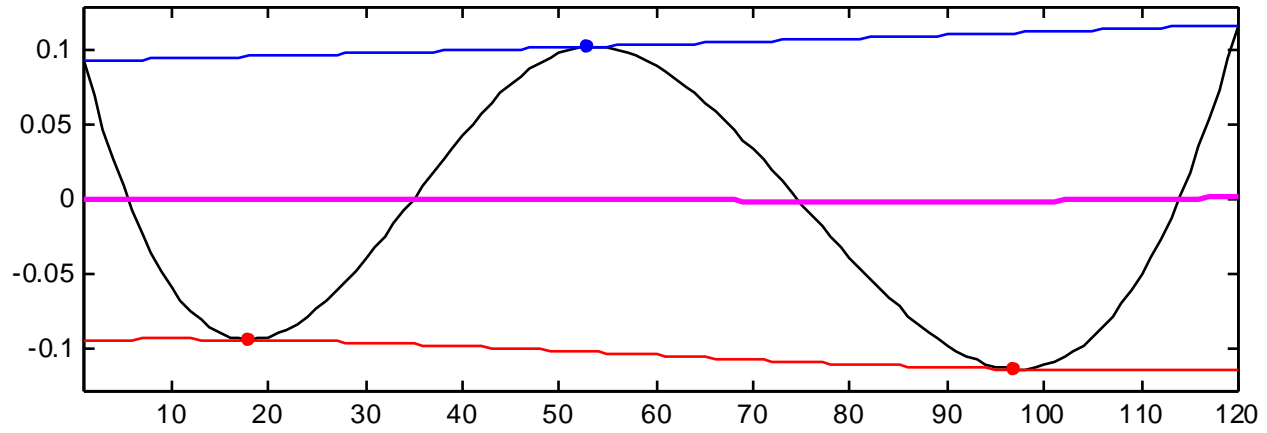
IMF 5; iteration 8



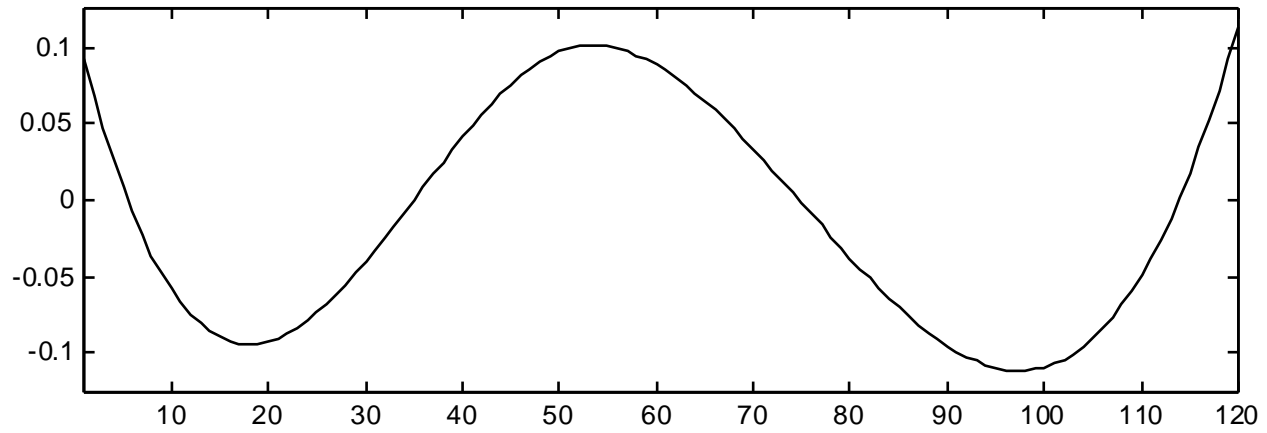
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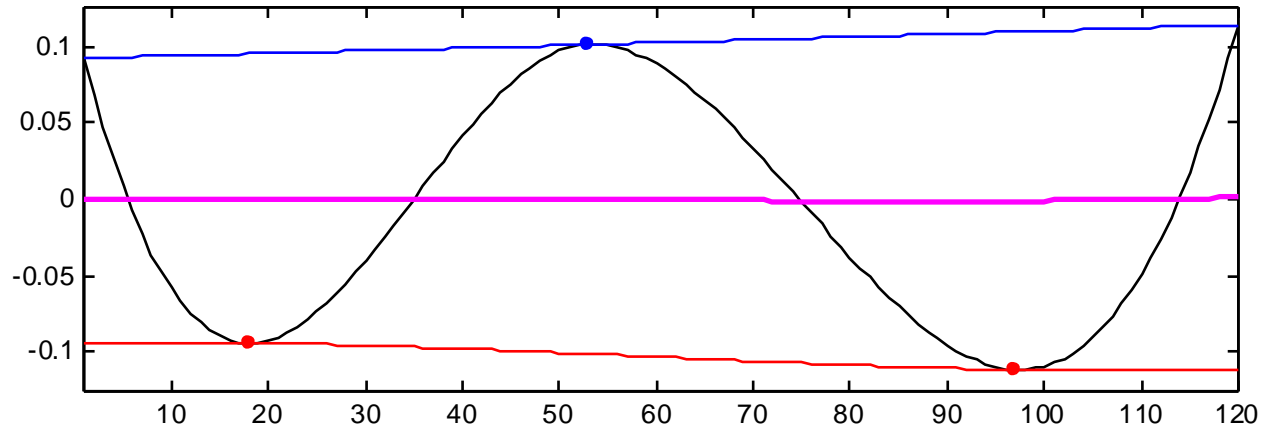
IMF 5; iteration 9



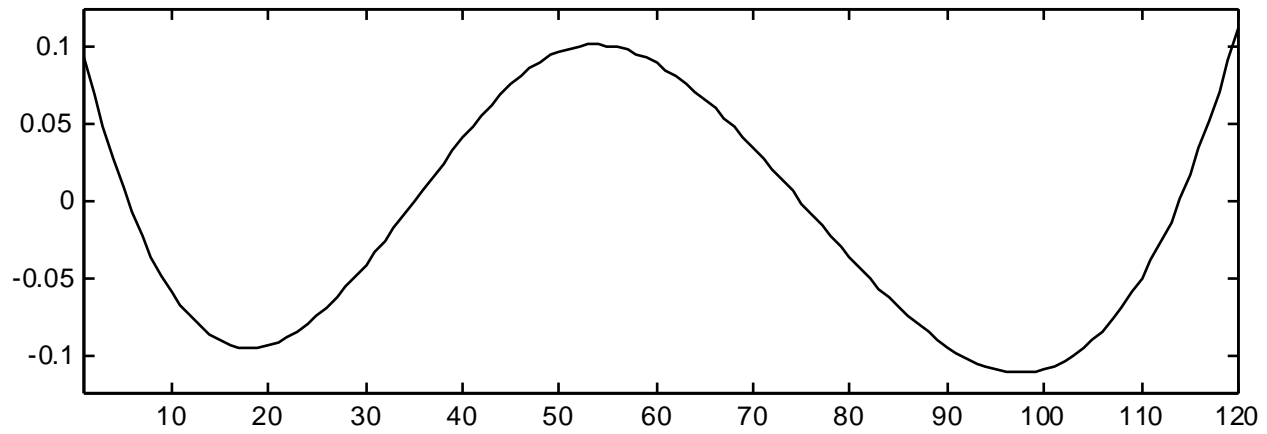
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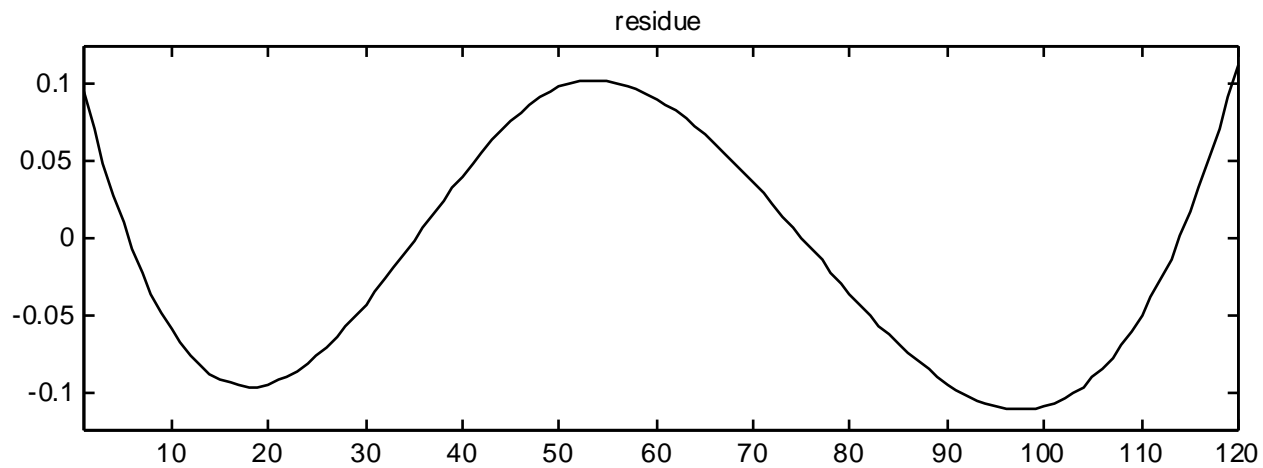
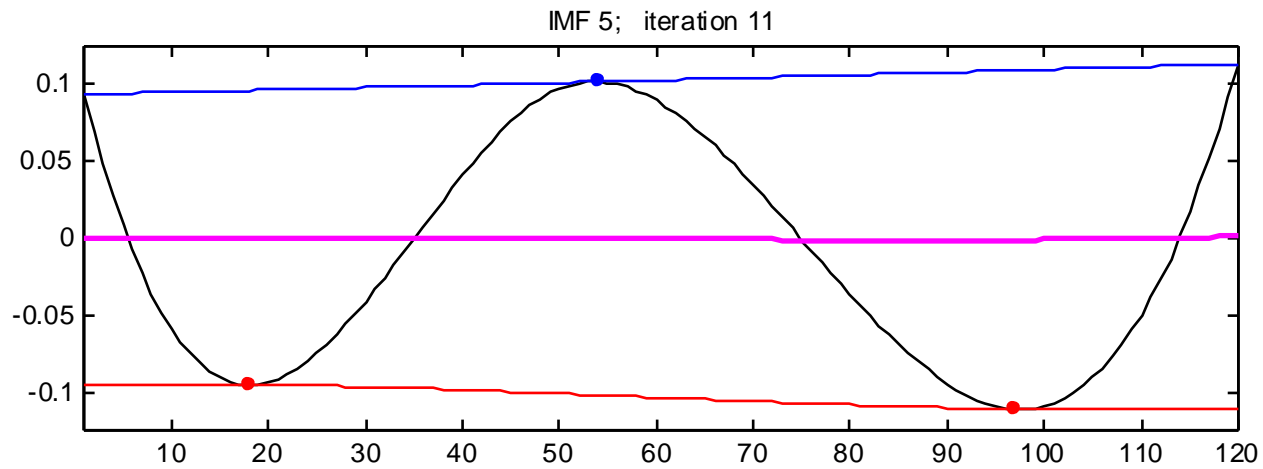


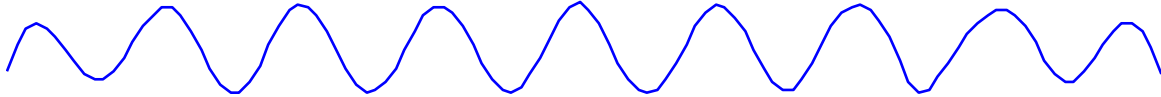
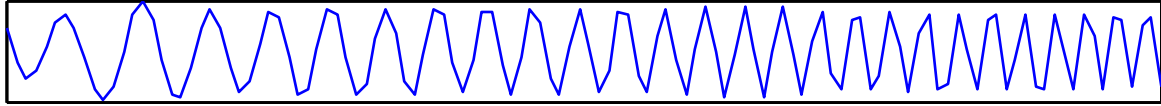
IMF 5; iteration 10



residue

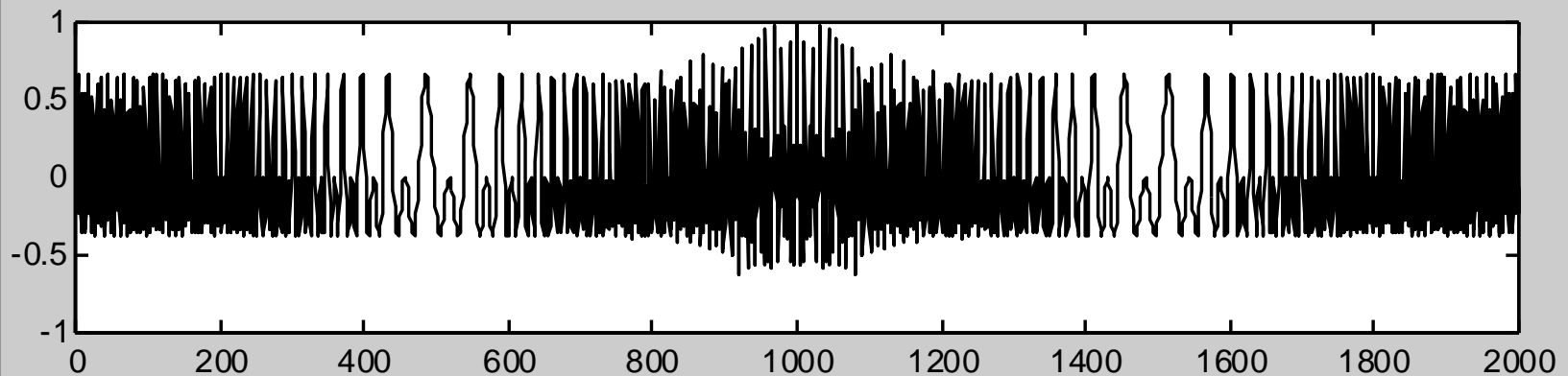




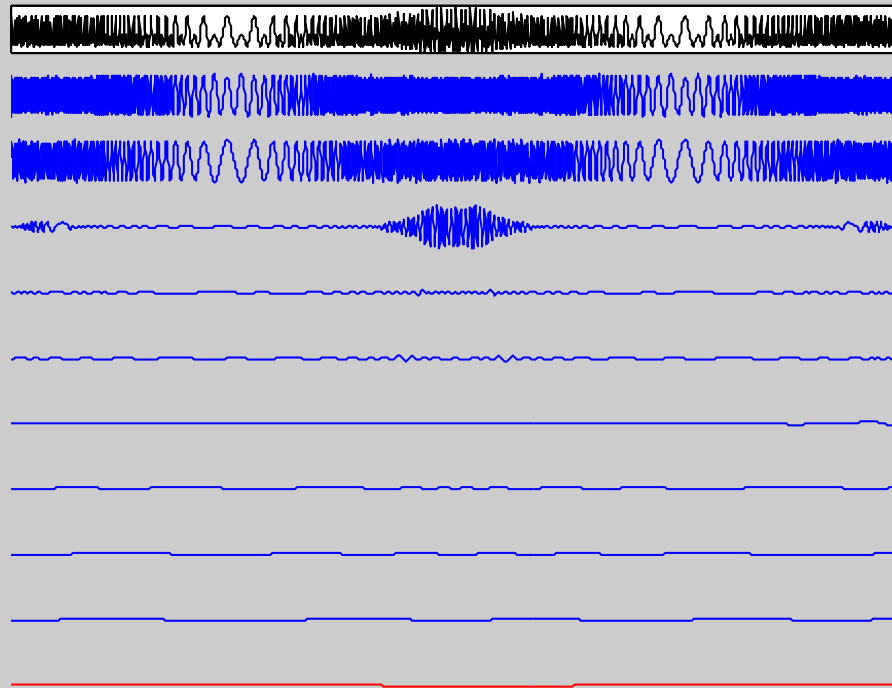


EMD and AM-FM signals

- quasi-monochromatic *harmonic* oscillations
- self-adaptive time-variant filtering
- example : 2 sinus FM + 1 Gaussian wave packet

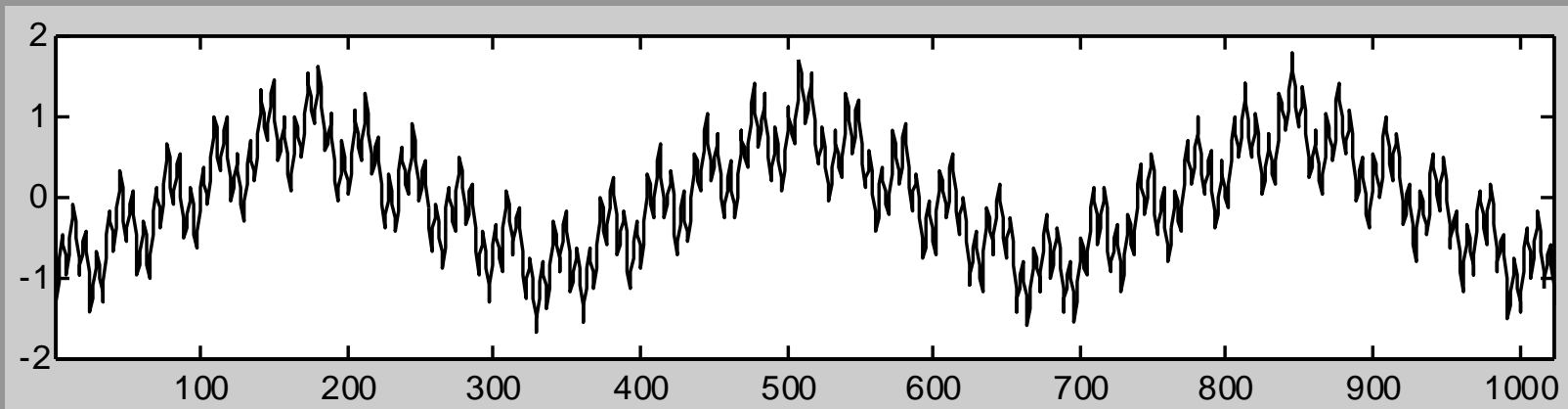


EMD

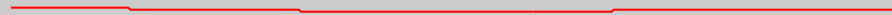
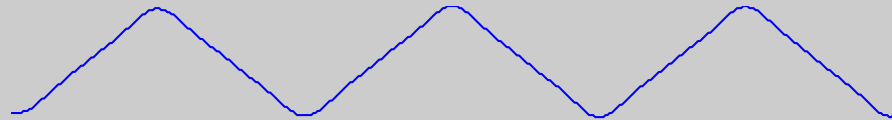
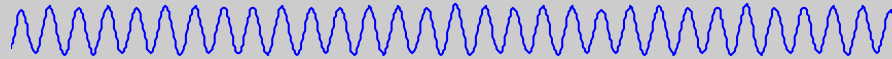
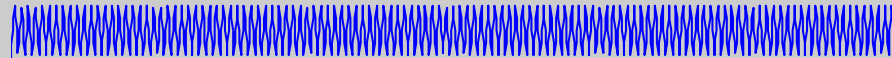
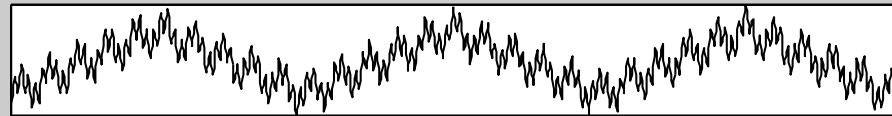


nonlinear oscillations

- IMF \neq Fourier mode and, in nonlinear situations, 1 IMF = *many* Fourier modes
- example : 1 HF triangle + 1 MF tone + 1 LF triangle



EMD



issues

- algorithm ?
 - intuitive but *ad-hoc* procedure, not unique
 - several user-controlled tunings
- performance ?
 - difficult evaluation since no analytical definition
 - numerical simulations

algorithmic issues

- **interpolation**
 - type ? *cubic splines*
 - border effects ? *mirror symmetry*
- **stopping criteria**
 - mean zero ? *2 thresholds*
 - variation 1 : « *local EMD* »
- **computational burden**
 - about $\log_2 N$ IMF 's for N data points
 - variation 2 : « *on-line EMD* »

performance evaluation

- extensive numerical simulations
- deterministic framework
 - importance of sampling
 - ability to resolve multicomponent signals
- a complement to stochastic studies
 - noisy signals + fractional Gaussian noise
 - PF *et al.*, *IEEE Sig. Proc. Lett.*, to appear

concluding remarks

- EMD is an appealing *data-driven* and *multiscale* technique
- *spontaneous* dyadic filterbank structure in « stationary » situations, stochastic (fGn) or not (tones)
- EMD defined as output of an algorithm: *theoretical framework* beyond numerical simulations?